An iterative approach to remove the influence of light ray bending from micron-scale scattered light tomography

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ABSTRACT
We present numerical experiments to test an iterative approach which can remove refraction-induced errors form scattered light tomography. Gradient scattered light tomography is the first method capable of direct non-destructive residual stress measurement in chemically strengthened glass. The method is based on an oblique incidence scattered light photoelasticity combined with an iterative approach to remove the influence of light ray bending from the scattered light method. In this article further numerical experiments are performed which demonstrated that the iterative approach grants the removal of the influence of light ray bending from stress profiles that have complicated shapes. Two classical examples of stress profiles in chemically strengthened glass were studied: (1) unusual stress profile, where the outermost layers of the surface are in tension (tensile surface stresses being 300 MPa), rather than compression; (2) relaxed compressive stresses in the outermost surface layer with a depth of 20 µm. We found that the nature and rate of convergence of the iterative process were notably different for compressive and tensile stresses. The analyzed stress profiles were taken from literature and hence are realistic representations of the ones that researchers might come across in glass science. Refraction induced reconstruction error $\Delta\sigma$ as a function of surface stress and incidence angle were simulated.

1. Introduction

Recently a non-destructive gradient scattered light method for micron-scale stress profile measurement in chemically strengthened glass was presented [1]. It was the first study that took into account the influence of light ray bending in the classical oblique incidence scattered light method. In case of strongly refracting media, such as chemically strengthened glass, where the component of the refractive index gradient $Vn$ is normal to the direction of light ray propagation, straight line inversion may result in a significant reconstruction error. Consequently, inversion schemes which incorporate correction for refraction need to be developed.

Chemically strengthened aluminosilicate glass [2,3], which has high scratch and impact resistance [4], has been used as a protective cover of space solar panels [5], mobile devices [6] and mirror foil for future X-ray telescopes [7]. Varshneya et al. [8–10] have contributed to the development of chemically strengthened lithium aluminosilicate glass, which has found its application as one layer in bulletproof armour plates due to its high compression stress (with a magnitude of $\sim$1000 MPa) and large case-depth (up to 1000 µm). Shim et al. have studied the impact resistance of chemically strengthened borosilicate [11] and soda-lime glasses [12]. They demonstrated that those glasses can be used as lightweight bulletproof materials. Residual stress profiles in all these strengthened glasses can be measured by gradient scattered light method, which emphasizes the importance of further studies on the theme.

In tomography, interferometric data is interpreted assuming that the probing light rays are straight lines within the medium under study. This simplified theory has been extremely useful if the refractive bending of light rays is small but can result in significant reconstruction errors in case of strongly refracting media. Therefore, reconstruction algorithms have been developed which take into account the bending of light rays. Dolovich & Gladwell [13] examined iterative schemes for reconstructing refractive-index fields to establish sufficient conditions for convergence. Lira and Vest [14] reviewed iterative approaches, intended for refractive index reconstruction, for which convergence is not guaranteed. In numerical experiments where these algorithms have been applied to data produced numerically from a known field, iterates have often diverged from the known field after initially approaching the reference solution. Vest [15] showed that for strongly refracting axisymmetric objects, reconstruction from optical path length data has small errors only near the axis. Acosta et al. [16] presented an iterative tomographic algorithm to reconstruct refractive-index profiles for fibre preforms and GRIN lenses from the measured deflection angles of

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refracted rays.

All other previous studies on the influence of light ray bending in residual stress tomography concentrated on transmission photoelasticity. The first studies about the influence of light ray bending in transmission photoelasticity were conducted by Bokstein [17]. Hecken and Pindera [18] and Aben et al. [19,58] suggested incorporating the experimentally measured deflections of light rays into the photoelastic theory. Dolovich et al. [20] described an iterative approach to remove the influence of light ray bending from transmission photoelasticity. They studied specific cases of a glass beam in pure bending and a diametrically loaded disc.

Pagnotta and Poggialini [21] studied how to compensate the influence of light ray bending in transmission photoelasticity in the case of residual stress measurement in axi-symmetric glass fibre preforms. They used experimentally measured radial refractive index profile by a P104 fibre preform analyzer (Photon Kinetics, USA) [22], rather than Maxwell stress-optic equations [23] that were used by Dolovich et al. [20]. The fibre was scanned with a He-Ne laser, and the deflection of the laser beam was analyzed to determine the radial refractive index profile. It was mentioned that a Mathematica™ program was developed, but neither the actual equations nor the complete simulation parameters were given that would allow recreating the presented simulations. However, experimental and calculation results (optical retardations along the curved ray path and straight ray path) were well described. For comparison, let us point out here that maximum axial stress levels in fibre preforms and fibres are in the range of 10−110 MPa [21,24] and surface compressive stresses for chemically strengthened glass up to −1000 MPa [1,9].

On the one hand, bending of light rays might be a source of errors in photoelastic tomography. On the other hand, refraction-induced optical retardation (integrated gradient photoelasticity [25]) can be an indicator of the stress profile. Although, this method is limited to stress profiles with known shapes, such as almost parabolic stress profile in thermally tempered glass plate. The precise shape of the stress profile in thermally tempered glass can be described by Indenbom’s [26] or Narayanaswamy’s [27] theory. Stress profiles in chemically strengthened glass plates can have very complex engineered shapes [28], which are not measurable by integrated gradient photoelasticity.

The possibility of scattered light method was already predicted by James Clerk Maxwell [23] in 1853 but experimentally discovered by Weller [29] in 1941. The scattered light method for stress profile measurement in tempered glass plates has been further studied by many authors [1,30–37]. Introduction of the laser as a light source for the method was made by Bateson et al. [31]. Cheng contributed to the development of oblique incidence method [32] and added dual observation technique [33]. Oblique incidence method for stress profile measurement in tempered glass plates was automated by Anton [34]. Gradient scattered light method [3] is specifically intended for measurement of micron-scale stress profiles in chemically strengthened glass. It is a new development from Anton’s polariscope SCALP by changing the incidence angle of the light beam from 45° to 81.9° and adding an iterative approach to remove the influence of light ray bending, which is the direct result of using such a high incidence angle. Hödemann et al. [35] introduced confocality as a detection method of Rayleigh-scattered light in order to reach ultra-high spatial resolution. Confocal mapping of a line along the laser beam propagation direction, using a micro-translation stage, is equivalent to the observation of a very narrow light ray (with diameter 5 μm) passing through the glass. Reviews on the scattered light method, in general, have been written by many authors [36–38], recent ones by Ramesh et al. [39] and Hemsley [61].

In this article, numerical simulations are performed to test the effectiveness of the iterative approach in the removal of the influence of light ray bending from realistic stress profiles with, e.g., tensile surface stresses or relaxed compressive stresses just under the surface. The aim is to find out whether an iterative approach grants a reconstruction of the stress profiles that have such complex shapes. The analyzed stress profiles were taken from literature and hence are realistic representations of the ones that researchers might come across in glass science.

2. Gradient scattered light method

The main concept of gradient scattered light method [1] incorporates ray tracing into the theory of oblique incidence method [32,34] to obtain the correct stress profile. Straight ray inversion is regarded as a first approximation to the residual stress depth profile. It is used in conjunction with the theory of gradient scattered light method to determine the first estimate of light ray paths and a curved ray inversion may be performed to yield (what is hoped to be) an improved approximation to the stress field. Then this approximation is used, in turn, to recalculate the ray paths which form the basis for the next iteration until a suitable termination criterion is satisfied. This recursive process is the basis of the gradient scattered light method.

2.1. Oblique incidence scattered light method

The method is based on Rayleigh scattering excited by a polarized beam of light passing obliquely through a flat chemically strengthened (or thermally tempered) glass plate (Fig. 1). Refractive index fluctuations, that are small in comparison to the light wavelength, act as dipoles that scatter the light predominantly in the direction perpendicular to the dipole axis. For the theoretical formulation of the method, we assume that the global x-axis lies in the incident plane and is parallel to the top surface of the glass whereas the global y-axis is perpendicular to the top surface of the glass and points into the glass. The global z-axis is chosen to form a left-handed triad with x and y axes. A local rectangular coordinate system x′-y′-z′ is associated with the light ray, such that x′ is tangent to the ray, y′ is on the incident plane and z′ is perpendicular to the incident plane. The clockwise angle between the x′ and y axes is denoted θ.

Secondary principal stresses (effective principal stresses) are defined as the stress components that are perpendicular to the propagation direction of the light ray. Those stresses induce Rayleigh scattered light fringe pattern that is observable in the direction OD1 and OD2. Principal stress σx is fixed in the direction of the optic axis and is independent of the light ray propagation direction. If the light ray propagates through the glass perpendicular to the z-axis, birefringence is caused by secondary principal stresses σy and σy, as seen in Fig. 1. The stress field defined in the global set of coordinates can be expressed in the local system via the stress transformation equations:

![Fig. 1. The experimental set-up for stress profile measurement. A laser beam is passed via a coupling prism obliquely through the strengthened glass plate. Ultrathin layer of immersion liquid is used between the prism and the glass surface to allow unimpeded propagation of light. From the observation directions of OD1 and OD2, that are at 45° to the surface of a glass plate, the scattered light intensity distribution along the light path could be recorded.](image-url)
\[ \sigma_x = \sigma_c \sin^2\theta + \sigma_s \sin^2\theta + 2\tau_s \sin\theta \cos\theta, \quad (1a) \]

\[ \sigma_y = \sigma_c \cos^2\theta + \sigma_s \sin^2\theta - 2\tau_s \sin\theta \cos\theta, \quad (1b) \]

\[ \sigma_z = \sigma_c = \sigma_s, \quad (1c) \]

where \( r \) is shear stress. Inside of the glass, all tensile and compressive stresses are parallel to the surface of the glass and depend only on the depth \( y \): \( \sigma_y = 0 = \sigma_x \). Away from the edges and in the case of uniform strengthening \( \sigma_c = 0 = \sigma_x = \tau_s \), because glass can freely swell in the direction of the \( y \)-axis and stress relaxation occurs.

Hence in case of isotropic stresses parallel to surface we can rewrite:

\[ \sigma_x = \sigma(y) \sin^2 \theta, \quad (2a) \]

\[ \sigma_y = \sigma(y) \cos^2 \theta, \quad (2b) \]

\[ \sigma_z = \sigma_s(y). \quad (2c) \]

Secondary principal stresses \( \sigma_x, \sigma_y \) and \( \sigma_z \) determine the refractive indices for light propagating in the \( x' \)-direction by Maxwell’s stress-optic relations \[23\]

\[ n_x - n_a = C_1 \sigma_x + C_2 (\sigma_x + \sigma_y), \quad (3a) \]

\[ n_z - n_a = C_1 \sigma_z + C_2 (\sigma_x + \sigma_y), \quad (3b) \]

where \( n_a \) is the refractive index for the e-ray, \( n_x \) is the refractive index for the o-ray and \( n_z \) is the refractive index of unstressed glass. Instead of being just a constant, the refractive index \( n_a \) have compositionally induced depth dependence.

The parameters \( C_1 \) and \( C_2 \) are the absolute stress-optic coefficients of the material for the considered wavelength and their difference gives the photoelastic constant (stress-optic coefficient):

\[ C = C_1 - C_2. \quad (4) \]

The stress-optic coefficient \( C \) is influenced by the glass composition. Nissle & Babcock \[40\] and Matsushita et al. \[41\] have published experimental results concerning system Na_2O-Al_2O_3-SiO_2 and found that Na_2O causes \( C \) to decrease whereas Al_2O_3 causes \( C \) to increase. Smedskjaer et al. \[42\] noted that different alkaline earth ions in aluminosilicate glass have different influences on \( C \) but all of them inflict a larger impact than Al_2O_3. In the case of chemically strengthened glass, there is a compositional depth profile, which also implies that \( C \) has a depth profile. For example \[35\], from experimentally measured (energy-dispersive X-ray microanalysis) Na\(^+\) ion concentration profile the Na_2O concentration profile was calculated and by using linear regression model the depth dependence of \( C \) was obtained.

Explicit expressions of \( n_x \) and \( n_z \) for a given stress distribution \( \sigma(y) \) are obtained by combining equations (2) and (3):

\[ n_x = n_a + \sigma(y) [C_1 \cos^2 \theta + C_2 (1 + \sin^2 \theta)], \quad (5a) \]

\[ n_z = n_a + \sigma(y) [C_1 + C_2]. \quad (5b) \]

Wertheim law in integral form combined with Eq. (5b) yields the optical retardation along a straight line segment and can be written as

\[ \delta(x') = C \int_0^{x'} (\sigma_z - \sigma_x) dx' = C \int_0^{x'} (\sigma_x - \sigma_s \cos^2 \theta) dx'. \quad (6) \]

The integral Wertheim law is a generalization of the classical Wertheim law for the case where stresses along the path of the light ray are not constant.

The incident laser beam not only induces the Rayleigh scattering but also a yellow fluorescence \[60\] that has to be removed using an optical filter. Consider an incident light beam that is linearly polarized parallel to the observation direction OD_1. The scattered light intensity, consisting of Rayleigh scattering and fluorescence, observable in the directions OD_1 and OD_2 can be written as:

\[ I_{001}(x') = e^{-k z} I_0 \frac{A}{k z} \left( \frac{2 \delta(x')}{2} \right)^2 + I_s e^{-k z}, \quad (7a) \]

\[ I_{002}(x') = e^{-k z} I_0 \frac{A}{k z} \left( \frac{2 \delta(x')}{2} \right)^2 + I_s e^{-k z}, \quad (7b) \]

where \( I_0 \) is the incident light intensity, \( I_s \) is the intensity of the fluorescence, \( A \) is the proportionality factor describing the probability of Rayleigh scattering and signal collection efficiency, \( \lambda \) is laser wavelength and \( k \) is extinction coefficient. According to Fourney & Chang \[60\] the component \( I_s \) is considerably larger than the Rayleigh scattered light. Fluorescence is spectrally broad and red-shifted compared to the incident wavelength. For example, if the exciting light is blue then fluorescence is usually yellow.

Scattered light intensity is usually experimentally recorded with a CCD camera \[34\] or confocally \[35\]. Scattered light polariscope SCALP transforms a scattered light intensity distribution along the ray path into a depth profile of optical retardation. Hence, we can bypass scattered light intensity and take optical retardation as starting point of iterative approach development.

### 2.2. Ray tracing

According to Fermat’s principle, a light ray travels between two points along a path that takes the least time. In inhomogeneous media, the path of least time is not necessarily a straight line and light rays become curved. If the spatial dependence of the refractive index in the medium is known, then the trajectory of any light ray can be calculated by using the well-known ray equation

\[ \frac{d}{dx} \left( \frac{du}{ds} \right) = \nabla n, \quad (8) \]

where \( ds \) is the differential length along the ray and \( dr \) is the resulting change in the position vector. For a flat piece of uniformly strengthened glass, the refracted light ray is confined into the plane of incidence \( \psi = 0 \) and the ray equation can be written in global coordinates as

\[ n^2 \frac{d^2 \psi}{dx^2} = \frac{\partial n}{\partial y} \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]. \quad (9) \]

The solution \( \psi(x) \) of this equation gives the ray trajectory. The refractive index \( n = n(y) \) and its partial derivative \( \partial n/\partial y \) can be directly calculated by using (5a) or (5b) if the stress profile \( \sigma(y) \) is already known.

In order to numerically solve Eq. (9), which is a second order ordinary differential equation (ODE), we first convert it to a system of two first order ODEs, by defining new variables

\[ u_1 = \psi, \quad u_2 = \frac{dy}{dx}. \quad (10) \]

The resulting first order equations are

\[ \frac{du_1}{dx} = u_2, \quad (11) \]

\[ \frac{du_2}{dx} = \frac{1}{n} \frac{\partial n}{\partial y} \left[ 1 + u_2^2 \right]. \]

We use an explicit Runge-Kutta method Dopi5 \[43\] to numerically integrate the system (11).

### 2.3. Refraction on the surface

In order to accurately calculate the ray paths in the glass sample, refraction on the prism-sample interface must be taken into account. Typically, the refractive index of the prism \( n_{\text{prism}} \) and the incidence angle \( \alpha \) in the prism are known experimental parameters. However, the angle of refraction in the glass sample depends on the stress, which is
obtained as the end result of the calculations. Therefore, the refraction on the glass surface must be recalculated on each iterative step, using the approximate surface stress from the previous step. For the s-polarized ray, this is quite straightforward as the refractive index on the glass surface can be directly calculated from Eq. (5b), after which the Snell's law can be applied. For the p-polarized ray, the index of refraction on the glass surface also depends on the angle of refraction, which is the unknown. It has been shown in [1], that combining Snell's law with Eq. (5b) yields a cubic equation

\[ n_r^2 - [n_r + \sigma_{\text{surf}}(C_1 + C_2)]n_r^2 - \sigma_{\text{surf}}(C_2 - C_1)n_r^2 \cos^2 \theta_r = 0, \]  

which has one real solution \( n_r \), that uniquely defines the angle of refraction, when inserted into the Snell's law.

2.4. Optical retardation along curved ray path

Experimentally measured optical retardation \( \delta_{\text{ray}}(y) \) is equivalent to optical retardation along curved ray path \( \delta(y) \). The idea is to calculate the optical retardation from initially known stress profile \( \sigma(y) \) to simulate the realistic situation where light ray bending occurs.

One can consider the glass plate consisting of thin planar layers, where the light ray propagates approximately along a straight line within each layer so that the optical retardation distribution through each layer of known stress profile \( \sigma(y) \) can be written as:

\[ \delta_{\text{ray}}(y) = \frac{C \sin^2 \theta_i}{\cos \theta_i} \int_0^y \sigma(y) dy \]  

(13a)

\[ \delta_{\text{ray}}(y) = \frac{C \sin \theta_i}{\cos \theta_i} \int_0^y \sigma(y+y_i) dy \]  

(13b)

\[ \delta_{\text{ray}}(y) = \frac{C \sin \theta_i}{\cos \theta_i} \int_0^y \sigma(y+y_i) dy \]  

(13c)

Eq. (13) does not grant a smooth optical retardation distribution because integral is starting from zero for each layer. Therefore the total optical retardation \( \delta_{\text{ray}}(y) \), that takes into account the accumulating optical retardation in all previous layers, is written as

\[ \delta_{\text{ray}}(y) = \delta_{\text{ray}}(y) + \delta_{\text{ray}}(y) \]  

(14)

where \( \delta_{\text{ray}}(y) \) is calculated in a similar manner based on layer \( i = 2 \), etc. By stitching together optical retardations for all the straight line segments a smooth continuous optical retardation distribution is formed. A set of if-conditions refines optical retardation along the curved ray path:

\[ \delta(y) = \begin{cases} \delta_0(y), & y_0 < y < y_1 \\ \delta_1(y - y_i) + \delta_1(y), & y_i < y < y_2 \\ \vdots \\ \delta_i(y - y_i) + \delta_i(y), & y_i < y < y_{i+1} \\ \vdots \\ \delta_{n-1}(y - y_n) + \delta_{n-1}(y), & y_n < y < y_0 \\ \end{cases} \]  

(15)

2.5. Straight ray inversion (SRI)

Starting with (measured or simulated) \( \delta_{\text{ray}}(y) \), there are two different options to reconstruct the stress profile: (1) straight ray inversion that assumes rectilinear light ray path and uses one constant incidence angle for the entire path length; (2) curved ray inversion that uses different incidence angles \( \theta_i \) for each straight line segment. Stress profile from \( \delta_{\text{ray}}(y) \) can be calculated using Wertheim law in differential form:

\[ \sigma(y) = \frac{\cos \theta}{C \sin \alpha} \frac{d}{dy} \delta_{\text{ray}}(y). \]  

(16)

2.6. Curved ray inversion (CRI)

The goal is to remove the influence of light ray bending from the measurement results. The idea of CRI is to first calculate the straight ray path using SRI and as a result, obtain the first estimate for the stress profile \( \sigma_{\text{est}}(y) \). This is the only possible way at first because we do not possess any data about the actual stress profile. \( \sigma_{\text{est}}(y) \) provides the first chance to calculate the refractive index distribution and perform the ray tracing for the e-ray and o-ray. After a sufficient number of iterations, the procedure should converge to the actual stress profile.

To reconstruct the exact stress profile from \( \delta_{\text{ray}}(y) \), a set of different incidence angles are required. Curved ray inversion for stress profile calculation can be written as follows:

\[ \left\{ \begin{array}{ll} \frac{C \cos \theta_i}{C \sin \theta_i} \frac{d}{dy} \delta_{\text{ray}}(y), & y_0 < y < y_1 \\ \frac{C \cos \theta_i}{C \sin \theta_i} \frac{d}{dy} \delta_{\text{ray}}(y), & y_1 < y < y_2 \\ \vdots \\ \frac{C \cos \theta_i}{C \sin \theta_i} \frac{d}{dy} \delta_{\text{ray}}(y), & y_i < y < y_{i+1} \\ \end{array} \right. \]  

(17)

The full sequence of iterative steps to remove the influence of light ray bending scattered light tomography:

(i) Straight ray inversion: At first, Eq. (16) is used to obtain an initial estimate of the stress profile \( \sigma_{\text{est}}(y) \).

(ii) Ray approximation: Curved trajectories of the e-ray and the o-ray are calculated by using the previously obtained \( \sigma_{\text{est}}(y) \) in the ray equation. The effect of stress-induced refractive index on surface refraction is also taken into account.

(iii) First iteration of CRI: The stress profile is recalculated using Eq. (17), where the curved trajectory of the rays is taken into account.

(iv) Ray approximation: Curved ray paths are calculated from the stress profile found in the previous step.

(v) Second iteration of CRI: stress profile is calculated from \( \delta_{\text{ray}}(y) \) by using the ray trajectories found in the previous step.

The changes in \( \sigma(y) \) and in ray trajectories become smaller on each iteration as the algorithm converges. In a real world scenario, there is always some uncertainty in the measured retardation data, which sets a limit on the maximum achievable accuracy. Therefore, it is suitable to end the iterative process, when that limit is reached. The CRI method converges quickly and typically only a 3–5 iterations are needed.

Fig. 2 shows the iterative scheme for curved ray inversion. The input data can be either a numerically simulated optical retardation along the curved ray path from an initially known stress profile or an experimentally measured optical retardation.

2.7. Separation of absolute stress-optic coefficients \( C_1 \) and \( C_2 \)

Considering the difficulties in the experimental determination [44] of the constants \( C_1 \) and \( C_2 \), it was previously suggested [1] calculating them using a simplified way. According to the literature [34,52] \( C_1 \) is ~4.3 times smaller than \( C_2 \). Hence, both absolute stress-optic coefficients are calculated by only knowing the experimentally measured \( C \). By taking \( C = \) constant (\( C = -7.77\times10^{-7}\text{MPa}^{-1} \)) \( C_2 \) can be calculated from Eq. (4).

The value of \( C \) can be determined by many automated devices such as automatic polariscope AP-07 (Glasstress Ltd, Estonia) [45], automatic birefringence measurement device ABR-10A-EX (UniOpt Co., Ltd., Japan) [46] and ABR-22 (UniOpt Co., Ltd.) [47]. Yoshiro Tajitsu [47] collaborated with UniOpt to create a more complicated device PEL-3 [47], which enables measurement of \( C \) in case of different incidence angles in temperature controlled environment. However, as far as we know none of these devices have been reported to be able to determine the values of \( C_1 \) and \( C_2 \) separately.
3. Numerical experiments

3.1. Stress profile with tensile surface stresses

The stress profile depicted in Fig. 3, taken from Donald and Hill [48], exhibits an unusual behaviour where the outermost layers of the surface are in tension, rather than compression. Tensile stresses can be induced into glass surface by chemical ion-exchange process in which small ions from external source are exchanged for larger ions in the glass surface (the inverse of the normal chemical strengthening method). This phenomenon was first reported by Stewart and Young [49], and later by Ernsberger [50] as a mechanism for revealing the presence of surface flaws on glass. Donald & Hill were the first to measure such profile in case of chemical strengthening of glass. They accomplished the profile shown in Fig. 3 by tempering a lithium aluminosilicate glass (composition in mol% 49.39SiO2-9.9Al2O3-29.6Li2O-9.96MgO-1.15P2O5) for 49 h in NaNO3 salt at 385 °C. K+ ions inside a glass, having effective ionic radius 133 pm, are exchanged for smaller Na+ ions (98 pm), therefore tensile stresses are induced near the surface. The stress profile was measured by using Bradshaw's layer etching method [51]. The stress-optic coefficient, used in their calculations, was taken to be 2.75 Br, which was an estimated value from literature rather than determined experimentally for the glasses under investigation.

In our numerical experiment the stress profile in Fig. 3 was taken as the initially known stress profile. Numerical simulations were performed to find out whether such a stress profile could be reconstructed from the optical retardation data. The absolute stress-optic coefficients were calculated using simplified approach: \( C_1 = -7.77 \times 10^{-7} \text{ MPa}^{-1} \) and \( C_2 = -3.28 \times 10^{-8} \text{ MPa}^{-1} \). The refractive index of the coupling prism \( n_{\text{prism}} = 1.515 \), the refractive index of unstrained glass \( n_u = 1.515 \) and incidence angle \( \alpha = 81.9^\circ \).

3.2. Results

3.2.1. Optical retardation and ray paths

Fig. 3 shows a comparison between the optical retardation \( \delta(y) \) along the curved ray path (calculated using Eq. (17)) and the optical retardation \( \delta(y) \) (calculated directly from initially known stress profile with tensile surface stresses using Eq. (6)). The increasing difference between \( \delta(y) \) and \( \delta(y) \) is a direct result of light ray bending. At the entrance point \( \delta(y) = 0 \), but at the exit point (at depth y=260 µm) the difference has increased to \( \delta(y) - \delta(y) = 150 \text{ nm} \).

Fig. 4 depicts the curved paths of the e- and o-ray propagating through the same chemically strengthened glass plate. The residual stress profile has a tensile surface stress of 304 MPa. Light ray bending is almost un-noticeable near the surface, which illustrates the difficulties of direct experimental observation of the light ray bending, although the scattered light fringe pattern (and therefore measurable optical retardation) is affected.

Fig. 5 shows the deviation of the o-ray and e-ray paths from the straight ray path. At the entrance point on the surface, the light rays deviate from the straight ray path in the positive direction (up to 7 µm) because the tensile stresses lower the refractive index.

3.2.2. Iterative approach

Fig. 6 illustrates how SRI and the first iteration of CRI deviate from the true stress profile. By applying SRI to \( \delta(y) \), a stress profile \( \sigma_{\text{sri}}(y) \) is obtained with a surface stress value of 327 MPa whereas the assumed stress profile had a tensile surface stress of \( \sigma_{\text{surf}}=304.5 \text{ MPa} \).

By using the first iteration of CRI (step (iii) in the iterative sequence) the resulting surface stress is 302.7 MPa (see Fig. 7(a)), which is much closer to the true value, but still wrong by 1.8 MPa. The 3rd iteration of CRI gives a surface stress of 304.7 MPa (see Fig. 7(a)), which is only 0.2 MPa wrong. Further iterations rapidly converge to the true value of 304.5 MPa. After the first iteration, the maximum error was reduced from 7.5% (SRI) to 0.6%.

Fig. 7(b) shows the convergence of the compressive stresses in the depth of 129 µm. SRI (0th iteration) gives ~305 MPa, which is wrong by 19 MPa, compared to the real value ~325.7 MPa. The first iteration gives ~324.5 MPa, with an error only 1.2 MPa. After the first iteration, the maximum error was reduced from 6% (SRI) to 0.4%.

Fig. 7 also illustrates that tensile and compressive stresses converge differently: tensile stresses oscillate about the real value whereas compressive stresses converge monotonically.
3.3. Stress profile with relaxed compressive surface stresses

The stress profile depicted in Fig. 8, taken from Donald and Hill [48], exhibits a relaxed stress profile, for which the maximum value of
compressive stress was found just under the surface, rather than at the surface. Donald & Hill produced the profile by tempering a lithium aluminosilicate glass (composition in mol% 50.81SiO₂-11.51Al₂O₃-17.68Li₂O-10.92MgO-1.03P₂O₅) for 49 h in KNO₃ salt at 385 °C. As Na⁺ ions inside the glass were exchanged for the bigger K⁺ ions, compressive stresses near the surface were induced.

The stress profile was measured using Bradshaw's method and a value of 2.75 Br was used for the stress-optic coefficient. Stress relaxation has been noted previously for both sodium aluminosilicate [51,52] and lithium aluminosilicate glasses [48,53]. It is associated with the effects of thermal stress relaxation [56,59] of glass when treatment is carried out at temperatures of the order of 100 °C or less below the strain point of the glass [54].

3.4. Results

3.4.1. Optical retardation and curved ray paths

Fig. 8 shows a comparison between the optical retardation δᵧ(c) along the curved ray path and the optical retardation δᵧ(s) along straight line path. At the exit point (at depth y=260 µm) the difference δᵧ(c) - δᵧ(s) has achieved the value of 16 nm.

Fig. 9 shows that at the entrance point the light rays tend towards the negative direction because the glass plate has a higher refractive index near the surface than the coupling prism.

3.4.2. Iterative approach

Fig. 10 illustrates how SRI and the first iteration of CRI deviate from the true stress profile. By applying SRI to δᵧ(c), a stress profile σᵧ(sri) is obtained with surface stress 191 MPa (shown in Fig. 10) whereas the assumed stress profile had a tensile surface stress of σᵧ(surf) = 199 MPa.

Fig. 10 shows that maximum SRI construction error is σᵧ(surf) = 297 MPa (at depth 19 µm marked with “a”). Fig. 11(a) shows convergence of compressive stresses at that depth and the first iteration of CRI results in a stress value of 317 MPa (see Fig. 11(a)), which is already quite close to the true value of 317 MPa. After the first iteration, the maximum error was reduced from 6% (SRI) to 0.4%. Further iterations rapidly and monotonically converge.

Fig. 11(b) shows similar iterative steps for reconstructing the compressive stress at the depth of 265 µm. After the first iteration, the error was reduced from 2.5% (SRI) to 0.06%.

3.5. Iterative approach in case of tensile and compressive stresses

In previous sections, it became apparent that compressive and tensile stresses converge differently. To further investigate this phenomenon we conducted an additional numerical experiment. We consider two stress profiles with identical shape, the only difference being interchanged tensile and compressive stresses.

3.5.1. Simulation conditions

A model stress profile through the thickness of a chemically strengthened glass can be described by

\[ \sigma(y) = \sigma_{surf} D^{-m} \left[ \frac{D}{m+1} \right] \]

(18)

where D is half-thickness and m is polynomial order. Eq. (18) was formulated in such a way by Brodland and Dolovich [57] in order to produce symmetric, self-equilibrating stress distributions, like those that typically arise in a uniformly (from both sides) strengthened glass plate. By increasing m in Eq. (18), the shape of the simulated stress profile becomes more characteristic to chemically strengthened glass.

The stress profile depicted in Fig. 4 was simulated using Eq. (18)
with polynomial order $m = 10$, and plate thickness 200 µm. The magnitudes of compressive and tensile surface stresses were both chosen to be 850 MPa. The absolute stress-optic coefficients were taken to be $C_p = -7.77 \times 10^{-7}$ MPa$^{-1}$ and $C_u = -4.077 \times 10^{-6}$ MPa$^{-1}$ [55], which yields to stress-optic coefficient of $C = 2.57$ Br, refractive index of the coupling prism $n_{\text{glass}} = 1.515$, refractive index of unstressed glass $n_u = 1.51$ and incidence angle $\alpha = 81.9^\circ$.

3.5.2. Results

Fig. 12 shows a comparison of the calculated stress as a function of iteration number. For tensile stress reconstruction after the first iteration, the maximum error was reduced from 26% (SRI) to 9%. For compressive stress reconstruction after the first iteration, the maximum error was reduced from 15% (SRI) to 2%. Compressive surface stresses converge $k = 1.7$ times faster than tensile surface stresses (Fig. 13).

Fig. 12 also illustrates that tensile and compressive surface stresses converge differently: tensile stresses oscillate about the real value whereas compressive stresses converge monotonically. Such different behaviour can be explained as follows. In the tensile stress case, SRI (0th iteration) gives an initial surface stress value of 1075 MPa. This value is used in the first estimation of refractive index on the surface. An increasing tensile stress causes a decreasing refractive index (Fig. 14(a)), therefore near-surface light ray propagation angles $\theta_i$ are corrected towards higher values ($\theta > \alpha$). First iteration of CRI gives surface stress that is calculated using higher values of $\theta_i$. According to Eq. (17), this leads to a new tensile stress value of 775 MPa. This in turn results in higher refractive indices and lower values of light propagation angles. Second iteration yields a surface stress value of 870 MPa, which leads once again towards higher values of $\theta$ than in previous iteration. Thus the calculated tensile stresses oscillate about the real value.

In contrast, an increasing compressive stress (by absolute value) causes an increasing refractive index (Fig. 14(b)). For compressive stresses the iterative approach corrects the refraction angle only towards lower values, therefore those stresses converge monotonically.

4. Limitations to oblique incidence method

4.1. Spatial resolution

In order to achieve a spatial resolution high enough to measure the stress profile in a chemically strengthened glass, a narrow laser beam is passed through the surface layer of the glass at a considerably large incidence angle of 81.9° [1]. This way, the increased length of the laser beam path becomes observable. For example, if a laser beam is passed at an incidence angle of 77.9° through the 200-µm-thick surface layer of a glass plate (Fig. 15), then the light travels a path length of 954 µm and the bending of light is seemingly almost non-existent. Although same rays are magnified in Fig. 16 where it can be seen that light rays curve and deviate from straight ray path by 0.7 µm. If the incidence angle is increased to 81.9°, the path length observable by camera...
becomes 1418 µm. That results in up to ~1.5 times increase in resolution.

4.2. Limitation to incidence angle

If we further increase the incidence angle, in a desire to increase the resolution, a spatial separation of the e-ray and o-ray occurs and the scattered light fringe pattern vanishes. Optical retardation distribution is measurable only if the scattered light intensity pattern is observable. This occurs only until the rays are spatially not yet totally separated, i.e., the separation distance is smaller than the beam diameter. This sets a limit for the incidence angle. Such a large incidence angle as 81.9° is very close to this limitation and hence the rays are on the verge of spatially separating and bending back to the surface.

Fig. 16 shows that in the case of a symmetric stress profile the entrance angle and the exiting angle have very close values due to symmetric nature of light ray bending. Fig. 16 illustrates that the light ray deviates from straight ray path by only 1.5 µm, which is less than the 50 µm diameter of an ultra-narrow laser beam.

5. Refraction induced error \( \Delta \sigma \) as a function of known surface stress

Fig. 17 illustrates refraction induced error \( \Delta \sigma \) as a function of known surface stress. The figure shows that if \( \alpha = 81.9^\circ \) then \( \Delta \sigma \) is < 5% for surface stresses < –465 MPa. In the case of \( \alpha = 77.9^\circ \) surface stresses up to –690 MPa are tolerable within this error margin.

In the case of higher incidence angle, a more rapid increase of \( \Delta \sigma \) is observable from Fig. 17, because the bending effect is strongest when light propagates perpendicularly to the refractive index gradient vector. From the experimental point of view 81.9° is very close to a unique incidence angle for which simultaneously the e-ray and o-ray are not yet spatially separated and the maximum length of ray path is observable. This so-called unique angle is not fixed because it depends on, along with other parameters such as absolute stress-optic coefficients, mostly on the magnitude of stresses. By further increasing the incidence angle a spatial separation of the e-ray and o-ray occurs and the observable scattered light fringe pattern vanishes thus it is not possible to measure optical retardation. Spatial separation of those rays is larger in greater depths meaning that experimental measurement of optical retardation can also have a depth limit.

6. Refraction induced error \( \Delta \sigma \) as a function of incidence angle

Fig. 18 illustrates refraction induced error \( \Delta \sigma \) as a function of incidence angle. One can see that \( \Delta \sigma \) is < 5% for incidence angles < 75.19° when surface stress is –850 MPa and < 85.5° if surface stress is –300 MPa. Fig. 18 assures that \( \Delta \sigma \) can be neglected in the case of low incidence angles.

7. Discussion

Straight ray inversion, which is a simplified theory that neglects
light ray bending, is an extremely useful way to calculate the stress profile from the optical retardation in a case of incidence angles such as 45°. However, scattered light method that would be capable of determining stress profiles in chemically strengthened glass requires a more complete theory to describe the interaction between light and birefringent media. We have shown that an iterative approach is capable of removing the influence from a stress profile with a complex shape and compressive and tensile stresses.

Dolovich et al. [20] pointed out that a reasonable goal is to establish a theory capable of removing the influence of light ray bending as a post-processing approach. We have demonstrated an iterative scheme that can remove the influence of light ray bending from scattered light method and fulfilled the Dolovich’s criteria of post-processing approach. This means that the measured optical retardation should be the only needed experimental data to complete the calculation that removes the influence of light ray bending from the stress profile. The post-processing approach suggested by Dolovich is a more elegant way to solve the problem, compared to methods relying on experimentally measured deflections of light rays.

In practical situations, it is usually much more convenient to measure just a stress profile, rather than both the deflections of separate light rays and stress profile. For example, in the case of transmission photoelasticity, the light that is passed through a sample is not a narrow laser beam but a wide collimated beam. In this case, an experiment with a separate light source (a narrow laser beam) is needed to scan sample and record the deflections of each exiting light ray. Pagnotta & Poggialini used P104 preform analyzer to conduct such experiment, but P104 is limited to measuring fibre preforms with minimum diameter 5 mm. Hence, for stress measurement in optical fibres, usually having a diameter around 100 μm, the use of Maxwell’s stress-optic equations may be the only option. We have shown that if a light ray passes through strengthened glass plate obliquely then the direct measurement of light ray deflections is restricted. Numerical experiments indicate that if refractive indices of coupling prism and strengthened glass substrate are equal then a light ray deviates from straight ray path by only 10 μm, which is less than the 50 μm diameter of an ultra-narrow laser beam.

Nonetheless, direct measurement of light ray deflections must not be completely disregarded because it can potentially have great value if the samples have a high compositionally induced gradient of refractive index. We must point out here that the iterative scheme only removes the stress induced influence on refractive indices. Lira and Vest [14] reviewed cases when iteraates diverged from the known field after initially approaching the reference solution. A qualitative explanation for this behaviour has been given in terms of discretization and round-off errors associated with the implementation of iterative algorithms. In our case iterates of both compressive and tensile stresses did not start to diverge after an initial approach to known stress profile. This can be explained by the fact that in case of transmission photoelasticity the reconstruction problem is highly nonlinear and small perturbations in initial conditions lead to large differences in the final result. In our case, the ray bending effect corresponds to a moderate correction term in the refractive index profile, which ensures that the system will remain stable even in the presence of numerical errors.

8. Summary

We have performed numerical simulations for two different residual stress profiles obtained from the literature. Presented theoretical results can be summarized as follows:

1) An iterative approach is suitable to remove the influence of light ray bending from the stress profiles with complex shapes.
2) Tensile and compressive surface stresses converge differently: tensile stresses oscillate about the real value whereas compressive stresses converge monotonically.
3) Compressive stresses converge faster than tensile stresses.
4) In all cases, the process converged before the fifth iteration when experimental errors were not taken into account.

References
