Sound Reflection from Subsonic Flow Duct Termination
Exhausting Hot Jet
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ABSTRACT
Noise radiation from a duct exhausting hot gas into relatively cold surrounding media is a classical problem in acoustics and a common issue addressed to engine exhaust outlets. The acoustic pressure reflection coefficient for a circular duct opening in hot gas conditions was recently investigated by the authors in case of negligible mean flow. It was found that the magnitude of reflection coefficient decreases significantly with increased temperature and does not exceed unity at Helmholtz numbers up to 0.8. In this study the purpose is to investigate the Mach number effects on the reflection properties of the flow duct opening exhausting hot jet. A dedicated test facility has been developed to perform the experiments with heated air flow (up to 180 °C and Mach no. M<0.31). The acoustic pressure reflection at the test duct termination is determined by following the classical two microphone technique. The experimental results obtained for the reflection coefficient magnitude and phase at subsonic flow velocities in hot conditions are presented and discussed.

1 INTRODUCTION
To comply with the tougher legislation the noise radiating from duct systems has become a significant problem. For the aeronautical application of a jet engine, the problem of sound radiation and reflection from a pipe termination in a uniform subsonic mean is an important issue. Other technological applications related are for instance tail-pipes of internal combustion engine exhaust systems, industrial duct openings and burner stacks. If noise transmission through the opening is to be estimated in these situations, knowledge of the acoustical boundary conditions at the duct opening is essential.

The reflection of sound by an interface separating two regions in relative motion was originally examined by Rayleigh [1]. In the classical theoretical work by Lewine & Schwinger [2] the problem of sound reflection at an and radiation from an open unflanged pipe termination has been solved for no flow condition by implementing the Wiener-Hopf technique. One of the early investigations to study the axial propagation of sound in the presence of mean flow through pipe opening was performed by Tsien [3] for exhaust nozzle of the rocket motor. A major analytical work to treat the sound radiation problem at the pipe opening has been carried out by Rienstra [4, 5]. An analytical model to solve the scattered sound field of a harmonic sound wave propagating out of an open ended annular duct was derived in [4]. In [5] a comprehensive analytical solution has been derived, by using the Wiener-Hopf technique, of the problem of sound radiation in and radiation from a semi-
infinite cylindrical pipe submerged in a subsonic coaxial uniform mean flow. In this work the energy balance of the acoustic field around duct opening was considered to evaluate the acoustic power loss of sound transmitted through a jet exhausting from a duct. It was found that for a low Helmholtz number plane wave the attenuation in uniform flow is of the same order of magnitude, increasing with the Mach number and with the hub diameter indicating that the cross-sectional surface is indeed a relevant parameter. It was also concluded that the variation of Mach number or a hub radius of the opening affects mainly the number of cut-on modes, and thereby the number of lobes in the far field radiation.

It was shown by Carrier [6] and Munt [7] that for the flow directed out of pipe opening the acoustic field sheds from the trailing edge a vortex sheet which has to be taken into account when estimating the sound field around the opening.

Theoretical analysis of the acoustic transmission and reflection for plane waves propagating from a moving medium, impinging on a plane shear discontinuity into a cold stationary region was done by Candel [8] to examine internal noise transmission through the exhaust duct of a turbojet engine. It was demonstrated that a temperature discontinuity has a major effect on the direction of propagation on the transmitted waves.

Miles [9] and Ribner [10] discussed the effects of the velocity discontinuity on the transmission, reflection, and the possibility of resonance, amplification and instability of the interface.

The influence of the Mach number on the reflection properties was extensively studied by Munt in [11, 12] using a linear theory and by including a Kutta condition at the pipe termination. It is important to mention that the Munt’s model was first to predict values for the plane wave reflection coefficient that exceed unity at low frequencies and M≠0, in accordance with the experimental observations in earlier investigations.

Measurements for acoustical damping and reflection coefficient for an open pipe at low Mach and Helmholtz numbers was performed by Peters [13] by implementing a multi-microphone approach. It was concluded that in the presence of mean flow the reflection coefficient and acoustical damping in a pipe are strongly influenced by the Mach number. By testing several duct opening geometries it was found that the magnitude of the reflection coefficient is not much influenced by the geometry of the termination, whereas the end correction showed significant influence.

Inspired by a good agreement with Munt’s theoretical model in a cold jet conditions reported in experimental works by Allam [14] and Ducret [15] a successful validation of the Munt’s theory was recently performed by Rämmal and Lavrentjev [16] for high temperature conditions. As the aim in [16] was to focus on the influence of the jet temperature on the sound reflection from the pipe opening and to exclude the flow effects, the flow velocity was kept negligible during these tests.

Despite of its frequent citation and the status of being often criterional for experimental and theoretical results in most of the contemporary publications on open-ended duct terminations (e.g. by Peters [13] and Allam [14]) the model of Munt [11, 12] has still only partly been validated. Only for the mean flow conditions up to M < 0.2 and k·a < 1.3 in case of air flow at room temperature and up to 500°C for hot conditions with negligible flow by the published experimental data [13-16]. It is reported in recent works (e. g. in [14]) that accurate measurements of the reflection coefficient for higher flow velocities, for hot gases or gases with different acoustic properties inside and outside the duct are of interest.

The purpose of the present investigation is to obtain experimental data for the reflection coefficient of a straight round pipe termination for a wide range of jet Mach numbers by introducing conditions where a hot jet exhausts into a relatively cold stationary surrounding media. Because of the super-imposed mean flow, the acoustics of such a duct system is more complex than for the no flow case. While providing quantitative data that can be used to
predict the acoustical performance of hot subsonic flow duct termination, the experimental data obtained can further be used to validate the Munt’s theoretical model for high temperature and Mach number conditions. In this paper the influence of the mean flow on the reflection properties for an open ended pipe exhausting hot jet is analyzed and discussed.

2 EXPERIMENTAL PROCEDURE

2.1 Experimental set-up

In current study the complex pressure reflection coefficient data are determined experimentally by following the classical two-microphone approach [17]. To perform the experiments with high flow velocities a dedicated hot flow installation has been set up. A sketch and a photo of the installation used to carry out the experimental work are presented in Figs. 1 and 2.

The measurements have been carried out in a test section made of steel tube of 1.2m length and inner radius of 41mm. The wall thickness of the tube was 2.0mm. The open pipe end as the primary test object was placed roughly in the middle of the laboratory hall (20m · 10m · 5m), 1.2m above the rigid floor and at 4m distance from the nearest wall.

As the acoustical test section of the pipe equipped with dynamic pressure and temperature sensors was always mounted more than 50 pipe diameters downstream of the stagnation chamber, a fully developed turbulent mean flow was expected during all the experiments.

A pressure transducer separation s = 0.07 m was chosen for the experiments. Boden and Åbom [18] proved that the two microphone method has its lowest sensitivity to errors in the input data in a region around ks < π(1 − M^2)/2. In [19] it was suggested that to avoid a large sensitivity to errors in the input data, the two microphone method should be restricted to the frequency range: 0.1π(1 − M^2) < ks < 0.8π(1 − M^2). Accordingly, the frequency region reckoned during the measurements ranged from 245Hz…1.9kHz (for room temperature air at 21°C) to 305Hz…2.4kHz (heated air at 180°C). The lowest cut-on frequency of this circular duct (f_c = 1.841 · c/(d · π)) was 4.9kHz.

To reduce the random error in the measurements with high flow velocities, a relatively large number of averages (1000 for all the test cases) was used.

Two piezoelectric pressure transducers (Kistler 4045A) equipped with water-cooling mounts were used in the test pipe to simultaneously record the acoustic pressure signals (see Fig. 1 and 2). The transducers were located on the opposite duct walls in order to suppress uncorrelated flow disturbances at the walls. The pressure signals were conditioned and amplified by piezo-resistive amplifiers (Kistler 4603). A random acoustic excitation was provided by electro-dynamic driver (DAS ND-8) placed into a side-branch of the test section (see Figure 1 and 2). The electro-dynamic driver was driven by a software-based signal generator through a power amplifier (Velleman VPA2100MN). The signal acquisition was performed by a four-channel dynamic signal analyzer (National Instruments NI PCI-4474). The analyzer was controlled by a purpose built PC based virtual instrument (LabView).

During the set of experiments with jet temperatures varied from the room temperature conditions up to 180°C, three flow velocities: v_1 = 50m/s, v_2 = 72m/s and v_3 = 108m/s (Mach number up to M = 0.31 in case of room temperature) were tested.

The flow velocity was determined by a portable anemometer (Delta Ohm) using a Pitot tube temporarily mounted to the duct opening.

In order achieve described flow velocities in the test section a 2 stage high pressure fan (Kongskilde 300TRV) was implemented. The fan was driven by a 22kW electric motor. The flow velocities were determined by regulating the speed of the motor via an electronic control unit. The outlet of the fan was directed into the inlet of a custom designed stagnation
chamber. The stagnation chamber, with a volume approximately $0.15m^3$, incorporated a 22.5kW electric heater unit (see Figs. 1 and 2) to raise the temperature of the air flow. In order to minimize the axial and radial temperature gradients in the duct, 5 surface heaters: four $550W$ heaters (Bencon 6009) and one $250W$ heaters (Bencon 5984) were mounted to the external surface around the test section. The temperature of both the internal and external heating systems was automatically set and maintained by implementing custom built control modules.

Six K-type temperature sensors were mounted to three different cross-sections of the duct. Temperature was measured in the both pressure transducer cross-sections and in the cross-section 10 mm from the duct termination. In each cross-section the radial positions of the temperature sensors were determined considering the equivalent flow areas in that cross-section. After the set temperature had stabilized in the test section the temperature values were obtained by a dual K-type thermometer (TES-1312). These temperature values were recorded and averaged in order to determine the actual speed of sound in the jet around the test section. The local temperature readings were furthermore taken into account to regulate the external heaters aiming to minimize the axial and radial temperature gradients.

![Figure 1: A sketch of the experimental set-up for the measurements.](image-url)
2.2 Determination of the reflection properties

The procedure used in this paper to experimentally determine the reflection coefficient of the duct opening follows the two-microphone method [17], which is shortly described below. The plane wave acoustic pressures at two pressure transducers separated by distance $s$ (see Figure 1) can be expressed as:

$$p_1(f) = p_{1+}(f) + p_{1-}(f),$$  \hspace{1cm} (1)

$$p_2(f) = p_{1+}(f)\exp(-ik_+s) + p_{1-}(f)\exp(ik_-s),$$  \hspace{1cm} (2)

where $p$ is the acoustic pressure, $f$ is the frequency, $k = 2\pi f / c$ is the wave number, $c$ is the speed of sound, - and + denote the pressure waves propagating in neg. and pos. direction relative to the $x$-axis, $k_- = k / (1 - M)$, $k_+ = k / (1 + M)$ and $M$ is the Mach number. The reflection coefficient at transducer 1 is defined as

$$R_1^f(f) = \frac{p_{1-}^e(f)}{p_{1+}^e(f)},$$  \hspace{1cm} (3)
where \( r \) denotes the values measured at transducer 1 cross-section and \( p_x^e, p_y^e \) is the acoustic field generated by the external source (electro-dynamic driver). From equations (1) – (3) it can be shown that

\[
R^e_x(f) = \frac{(H^e_{12} - \exp(-ik_s s))}{\exp(ik_s s) - H^e_{12}},
\]

where \( H^e_{12} = h_2 / h_1 \) is the transfer function from pressure transducer 1 to 2 and \( h_i = p_i / e \) and \( h_2 = p_2 / e \) are the transfer functions between the pressure transducer signals \( p_1, p_2 \), and the reference signal (here the electrical signal \( e \) driving the electro-dynamic driver), respectively.

The reflection coefficient \( R_s \), presented in the results section of this paper is finally transferred to the duct opening cross-section by

\[
R_s = R^e_x \exp(i(k_+ + k_-)l)
\]

where \( l \) is the distance from the transducer 1 cross-section to the duct opening.

3 RESULTS

The magnitude and the phase of the reflection coefficient for the pipe opening at two different subsonic flow velocities are presented in Figs. 3 and 4. In these figures the results from the experiments of varied jet temperatures are plotted for constant flow velocities in order to demonstrate the reflection coefficient dependence on the jet temperature. In Figs. 5 and 6 the temperature of the has been kept constant and the results for three different flow velocities are displayed to illustrate the effects of the Mach number. For a generalized representation of the results the figures are plotted in Helmholtz number \( (k \cdot a) \) domain.

From the Figs. 3 and 4 it is evident that the peak of the reflection coefficient magnitude in the critical \( k \cdot a \) range where it exceeds unity, is lower and less pronounced in case of higher temperatures. It can be additionally noticed that the reflection coefficient measured at higher temperatures has a tendency to decrease faster at increasing Helmholtz numbers and therefore causes a lower pressure reflection at the pipe opening in the higher \( k \cdot a \) range. The lower pressure reflection coefficient at the opening measured in the higher jet temperatures during the current high flow speed experiments confirms similar tendency reported in [16] for the hot no flow case.

In Figs. 5 and 6 a considerable reflection coefficient dependence on the jet velocities can be seen. It can be summarized that in case of higher jet velocities the reflection coefficient magnitude attains a higher value except in the lower Helmholtz number region. Therefore less acoustic radiation out of the pipe opening can be expected when the flow velocity is higher in a pipe exhausting a hot jet. This finding is also in a good correlation to the earlier results obtained for the room temperature conditions (e.g. in [12]).

For \( k \cdot a = 0 \) the value of the phase attains the value of \( \pi \) and decreases with increasing \( k \cdot a \). In case of higher temperatures (see Figs. 3 and 4) the phase of the reflection coefficient shows a significantly steeper decline compared to the room temperature condition tested. It should be noted here that for the cases of varied Mach number the phase does not exhibit a remarkable dependence on the flow velocities (see Figs. 5 and 6) tested in this paper.

For of all the jet conditions presented in Figs. 3-6 the magnitude of the reflection coefficient tends to increase with the flow velocity and exceeds unity at all the velocities tested. The maximum value around 1.2 is reached at the highest flow velocity tested (M=0.31
and 30°C). The values above unity have also been observed for the room temperature flow conditions tested up to $M=0.2$ by previous authors (e.g. in [12]). This phenomenon is apparent here in case of considerable flow velocity difference between the exhausting jet and the ambient gas. The hump of the reflection coefficient magnitude above unity has been explained in [12] by the transfer of acoustic energy into the kinetic energy in the shear layer in case of flow velocity difference adequate to cause vortex shedding in the shear layer. The current experimental results show that this phenomenon is continuously apparent at least up to the maximum velocity tested in this study ($M=0.31$).

Figure 3: The magnitude and the phase of the reflection coefficient for open-ended duct; Flow velocity: 72m/s; 30°C (blue circles), 100°C (black stars), 180°C (red squares)

Figure 4: The magnitude and the phase of the reflection coefficient for open-ended duct; Flow velocity: 108m/s; 30°C (blue circles), 100°C (black stars), 180°C (red squares)
4 CONCLUSIONS

A series of experiments to study the acoustical performance of a duct opening have been performed with various jet velocities and temperatures, reaching up to $M=0.31$ and $t=180^\circ C$ respectively. As the new insight into the acoustics of flow ducts, the tests were performed in conditions were both the velocity and the temperature of the jet were significantly varied and the consequent effects on the pressure reflection properties at the duct opening have been analysed and discussed.

It was found that the magnitude of reflection coefficient is strongly dependent on the jet Mach number and that it increases significantly with increased flow velocity by forming a characteristic hump above unity in a critical Helmholtz number region. The results obtained in this paper for hot conditions further confirm this behaviour, reported in earlier investigations for room temperature flow conditions. It was also noticed that at high flow velocities the magnitude of the reflection coefficient tends to decrease when the jet temperature is being increased. Similar trend had recently been reported by the authors for the no flow situation tested. In practice this result means that more acoustical energy (thus more noise) is being transmitted into the surrounding fluid from the pipe opening by the exhausting jet in case of higher temperatures.
An interesting continuation for the research would be to use the experimentally obtained data for the validation of the Munt’s theory in hot subsonic jet conditions. Also, it is of interest to perform such tests on different opening geometries that are often used for practical applications (bends, diffusers, oblique cuts).

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6 REFERENCES