Primary School Choice in Tallinn: Data and Simulations

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Abstract

Within the first 20 years of the market economy in Estonia, the public school market has been decentralized in Tallinn. Firstly, we describe how students are allocated to primary schools in a narrative, and secondly, in a formal mechanism design language. We indicate the closest equivalent algorithms from the matching markets design theory and conclude that the current system in Tallinn is a hybrid. The decentralized part of the market namely inter-district exam schools apply autonomous school proposing deferred acceptance; and the centralized part of the market intra-district regular schools apply school random serial dictatorship. Finally using data from Tallinn primary school matching, we show by an empirical evaluation of matching mechanisms. Since the currently the collected data does not reveal the students preference ordering we simulate some potential orderings for comparison.

Introduction

Economists have gained significant experience (and fame) in practical market design in the recent years. The applications of the theoretical principles of market design demonstrate that institutions matter at a level of details that economists have not often had to deal with. Moreover, there has been much research into allocating primary school spaces to students in primary (e.g. Abdulkadirolu and Sönmez 2003; Dur et al. 2013), secondary (Dur et al., 2013) and also high-schools (Abdulkadirolu et al., 2009). Much of this research is based on seminal papers by Gale and Shapley (Gale and Shapley (1962)) and were initially used for entry-level job markets such as National-Resident Matching Program and others (Roth (2008)). The core of the curricula is to apply a central mechanism that collects information from market participant as preferences and finds the allocation that has some merits.

The existing matching mechanism literature is growing, not only by new cases and designs, but also by adding new problematic design areas, i.e. encouraging diversity using quotas or priority classes that in many cases can fail to enforce social justice (Dur et al. (2013); Kominers and Sönmez (2013); Fragiadakis and Troyan (2013); Erdil and Kumano (2012)). However, to our best knowledge, there is no literature dealing with post-communist school allocation mechanism. While our experience indicates that mechanisms were widely in use in many spheres, i.e. assignments of university graduates or university choice. One common characteristic of communist mechanism was the school-proposing nature. However, central matching has been abandoned after the 90es during the liberal reforms and substituted by decentralised or semi-centralised designs. Our attempt is to contribute to this research gap by adding a case-specific description of the struggle that such systems go through by constant amendments to the allocation principles and the administrative urge for an improved (or politically justified) match. Moreover, we suggest that such school-proposing mechanisms are in use in many other post-communist countries, and revelations of the faults and merits of the current systems reveal
valuable policy implications not only for Tallinn, but also for other areas with similar systems in place.

As an empirical strategy, we naively compare the seat allocations of the Tallinn mechanism to other well-known mechanisms, such as the deferred acceptance and Boston mechanisms, by using data from the centralized database, e-kool (e-school). The e-kool database is an electronic register where approximately 4000 7-year old children with known home address list their school preferences annually. There was a somewhat special situation in 2011, when students could apply to any school in Tallinn (as intra-district policies were abolished). Before and after 2011, only inter-district schools have been privileged to accept student from other districts. Obviously the results can only be interpreted under the assumption of a truthful preference revelation, which might not always be the case.

As an empirical strategy, we naively compare the seat allocations of the Tallinn mechanism to other well-known mechanisms, such as the deferred acceptance, the top-trading cycles and the Boston mechanism, by using data from the centralized database, e-kool (e-school). The e-kool database is an electronic register where approximately 4000 7-year old children with known home address list their school preferences annually. There was a somewhat special situation in 2011, when students could apply to any school in Tallinn (as intra-district policies were abolished). Before and after 2011, only inter-district schools have been privileged to accept student from other districts. Obviously the results can only be interpreted under the assumption of a truthful preference revelation, which might not always be the case.

Our research strategy is as follows. First, we describe existing literature, highlighting the properties of the mechanisms, mainly stability, efficiency and strategy-proofness. We also list the popular mechanisms and causes of the recent changes in choice policies. The second section is entirely dedicated to Tallinn’s school choice mechanism, giving interpretive ground and case limitations, but moreover aiming at ‘translating’ administrative decisions and legal regulations into a mechanism design language. The following sections comprises the main body of the analysis, revealing the features of the current mechanism through examples, section 3, and empirical tests, section 4. Finally, we conclude by highlighting the policy implications for Estonia and for other decentralized markets.

1 Literature overview: Matching mechanisms and their properties

Our main aim is to reveal properties such as stability and efficiency in Tallinn’s allocation mechanism. For this, we first briefly review some school place allocation algorithms that are or have been used mostly in the US and UK, the most infamous being the Boston mechanism. The literature gives us insights into their good properties and the trade-off between them that centralized markets allow to gain.

1.1 Student-proposing Deferred-Acceptance

Deferred acceptance (DA) is the best-known principle (Roth, 2008), with many applications including school choice. Given student’s preferences over schools and school priorities over students’, the student proposing DA determines the match using the following procedure:

Round 1 Students apply to their first choice. Schools reject students not on their priority list and students who are over the school’s capacity. Assign others tentatively to the school.

Round k Students previously rejected apply to their second-choice school. New proposals and students on the tentative list are considered, and applications on schools’ priority list that
are not over school’s capacity are tentatively accepted. Reject others.

Terminate algorithm when there are no more proposals: i.e. all students are assigned to a school or rejected by all schools in their list.

The important properties of allocation are stability, strategy-proofness and efficiency. Stability means that (a) no student is preferred for a place over his current match in a school that also has a higher priority for the current student (such pairs are called blocking pairs), (b) every student weakly prefers his assigned seat to any remaining unassigned. Thus, the stable mechanism eliminates justified envy it has no blocking-pairs. Strategy-proofness means that the outcome is not vulnerable to manipulation, indicating that for each student, the dominant strategy is to state true preferences for schools; in other words, students cannot do better than by stating their true preferences. Efficiency means that allocation should promote students or schools’ welfare. In the case of Pareto efficiency, it is granted that an individual’s welfare cannot be increased without hurting others’. In most cases, it is specified whether the mechanism promotes only the student’s (or in peculiar cases only the school’s) welfare. In school choice, because of its properties, this popular mechanism is often referred as a student optimal stable mechanism (SOSM). The particularity of the SOSM is that it offers stable matching which is best for the students, meaning the proposing side (Abdulkadirolu and Sönmez, 2003; Roth, 2008)), but this does not mean that allocation is efficient. The latter points to the fact that, in recent literature, strategy-proofness or non-manipulability is the most referred desirable characteristic of matching mechanisms over alternatives such as efficiency and welfare gains (Ergin and Sönmez, 2006; Chen and Sönmez, 2006). However, some recent research also shows that, in some cases, Boston-type mechanisms over-perform SOSM (Abdulkadirolu et al., 2011; Featherstone and Niederle, 2011) in terms of efficiency without any considerable loss in stability. It is the case that more (Pareto) efficient matching can be found by breaking the stability requirement. For example, top-trading cycles allow Pareto improvements. This could be desirable and welfare enhancing when one side (schools) does not have a strict preference over the alternatives.

1.2 First preference First and Boston Mechanism

First preferences first (FPF) is a mechanism that gives priority to students according to their (parents’) stated preferences, and has been used mostly in England Pathak and Sönmez (2011). It is a hybrid between the DA (SOSM) and Boston mechanisms. The latter gained its name due to its being applied in Boston, MA, until 2005 (Abdulkadirolu and Sönmez, 2003). The Boston mechanism works as follows:

**Round 1** For each school, consider the students who have listed the school as their first choice. Assign these students to the school in priority order (e.g. based on distance and siblings) until no more places or students are available.

**Round k** For each school that has free places, consider students who have listed the school as their k-th choice. Assign these students to the school in priority order until no more places or students are left.

The procedure terminates when each student is assigned a place in a school or no more places remain available.

Besides Boston, this mechanism has been used in many US and UK school districts (Pathak and Sönmez, 2011). However, it has recently been abandoned in many school districts and substituted mostly by DA because of its strategic complexity and openness to manipulation Abdulkadirolu et al. (2006). The latter means that it discriminates against some students
because of how they ranked schools and thus makes them act strategically by misrepresenting their preferences. Moreover, Boston suffers from instability, as it can result in blocking pairs and may thus prevent the elimination of justified envy.

The mechanism used in England (FPF) is slightly different from the Boston mechanism. There may be two types of schools: for one set of schools, the allocation mechanism is DA, and for the rest, the Boston mechanism is used. This means it is a hybrid. The mechanisms are executed sequentially: first, students are allocated to DA schools, and the remaining students are considered for Boston mechanism schools. Thus FPF has the same strategic properties as Boston and in 2007 it was ruled illegal and substituted mostly by DA (Pathak and Sönmez, 2008).

Despite that the Boston mechanism has created a rule of thumb for manipulating the preferences (Pathak and Sönmez, 2008), there are still different levels of sophistication among families who participate in the mechanism. One possible school strategy is to avoid ranking two over-demanded schools as their top choices (ibid 2008: 1637). Recommendations were made to give a popular school as a first choice, plus a ‘safe’ second choice. As a result Pathak and Sönmez (2008) showed that the Boston mechanism is a coordination game among sophisticated students. Thus, ’levelling the playing field’ by diminishing the harm done to parents who do not strategize or do not strategize well is emphasized as a condition in designing an appropriate mechanism.

1.3 Top-Trading-Cycles and Serial-Dictatorship

One candidate for an alternative choice mechanism is top trading cycles (TTC), a mechanism that has been discussed in several contributions (Abdulkadirolu et al., 2006; Abdulkadirolu and Sönmez, 2013), that does not consider schools’ priorities as strict preferences. It allows for the option of two students with a preference for the other’s place to swap, regardless of the school’s priorities or preferences. It might be the case that a student is assigned to a school for reasons of proximity or because a sibling attends the school too, but the student would prefer to go to a different school. In that case, with TTC, the student is considered as having an option to exchange his seat with a student assigned to a different school, where the first student does not have such a high priority.

Moreover, TTC reduces to serial-dictatorship (SD) when all schools have the same priority ordering over students (Abdulkadirolu and Sönmez, 2003). SD will order students by some sort of procedure, mostly by test results or similar, giving the most able priority in the choice of his top preference. Abdulkadiroglu and Sönmez 1999 discuss the possibility of using random SD, which orders students using a lottery, and assigns them to their top choice, the next student their top choice among the remaining slots, and so on. This random SD is not only Pareto efficient, but also strategy-proof. That is, a truthful preference revelation is a dominant strategy for the student (or their parents). The limits in using such mechanism is based on local policies or community preferences, which often prioritize students according to distance from school or those whose siblings already attend the school. Therefore, it will be problematic to use a single lottery to create initial positions. However, it is possible to consider that some places within the schools may be allocated through random serial dictatorship. However, the student admission mechanism should be flexible enough to give students different priorities at different schools.

Then the guiding policy is to promote student welfare to the extent possible then TTC introduced by Abdulkadirolu and Sönmez (2003). According to TCC, initially every student and school is available:

Round 1 Create a graph where each student indicates her top-choice school and simultaneously, that school indicates their top-choice student. If there are cycles, assign students to schools in a cycle. Every student is part of at most one cycle.
<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Strategy-proofness</th>
<th>Stability</th>
<th>Pareto efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boston</td>
<td>No</td>
<td>No</td>
<td>No 1</td>
</tr>
<tr>
<td>First Preferences First</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Deferred-Acceptance (Student-Optimal)</td>
<td>Students</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Deferred-Acceptance (School-Optimal)</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Top-Trading-Cycles</td>
<td>Yes</td>
<td>No</td>
<td>Students</td>
</tr>
<tr>
<td>Serial-Dictatorship</td>
<td>Yes</td>
<td>No</td>
<td>Students</td>
</tr>
</tbody>
</table>

Round \( k \) Among remaining students and school places, create a graph where each student indicates her top-choice school and simultaneously that school points to their top-choice student. If there are cycles, assign students to schools in a cycle. Every student is part of at most one cycle.

The process terminates when all students are assigned or when there are no available places in any school.

TTC does not necessarily produce a stable matching. We can interpret stability as a proxy for eliminating justified envy. Thus, the policy choice here indicates the trade-off between efficiency and equity (elimination of justified envy). TCC (and SD) promotes student welfare while DA promotes equal treatment of students.

2 Tallinn School Market

We are interested in contributing to the many-to-one matchings in two-sided markets research. We investigate the specific cases or the microstructure of the decentralized and centralized markets. Moreover, we see that even in decentralized cases, the DA algorithm was discovered by trial and error Roth (2008). This may be due to the fact that DA captures the idea of how markets should operate. But this explains only one part of the Tallinn school market, specifically the oversubscribed inter-district schools, which are also not without problems. Admission to intra-district schools was centralized after 2011 and a version of RSD is applied to find a match.

2.1 Admission policy and choice reform in 2011

Before 2011, the distinction between inter- and intra-district schools became a widely accepted policy (see more detailed description in Pöder and Lauri 2014). Over the years, open enrolment schools have become increasingly over-subscribed. Thus all open enrolment schools have introduced aptitude entrance tests for the primary school intake (hereinafter exam schools). For intra-district admission schools (hereinafter regular schools), the tradition has been a central or semi-central catchment-based allocation based on an application (single preference) from the parent.

In 2011 the admission procedure was changed. The distinction between inter- and intra-district schools was abolished. All schools had autonomy over student intake, which most

\*\*\*Students don’t report actual preferences\*\*\*

5
often took the form of school specific aptitude tests. Latter means that in 2011 all schools could use preference orders over students, and parents (or students) would have the option of applying to whichever school they deemed best. While information about school preferences was not public, also requirements for the admission test were not completely transparent. The admission procedure was decentralized, but students still had to register their applications centrally because, in the event that they ended up without a place, final allocations would be made by the education commission.

After the applications were submitted to the schools, schools ran either aptitude tests or some sort of an interview with the students. This was independently organized and without any central guidelines. Based on their preferences, schools started making independent offers to students. Mechanism works as follows:

Round 1 Schools make offers to top students based on the admission-exam results. The student accepts his preferred offer and rejects the others. If the proposing school is not a first choice for the student, it is unclear if a student should accept, as a preferred place may open up in following rounds.

Round k Schools that have free places make offers to top students that have not already accepted or rejected them. The students accept an offer from their top choice and reject any previous offer they might have had.

Procedure is terminated when the end date is reached.

We propose that in 2011 the decentralized market applied a school-proposing DA (ScDA) that is optimal for neither student nor school for several reasons. Firstly, the decentralized nature of the aptitude tests makes students think strategically about the schools (total number and specific schools), as they have time constraints when taking admission exams (high opportunity cost of presenting truthful information). Secondly, due to the time constraints (on the application procedure) the algorithm may be considered to have terminated mid-run, indicating that it has not yet reached an optimal stable allocation. Thirdly, we may speculate that due to the nature of the school offers, the algorithm does not produce student-optimality (equivalently to SOSM).

2.2 Admission policies from 2012 onwards

From 2012 onwards, policy reverted back to the previous model where some exam schools had the option to state their preferences for students while for others (regular schools) students were assigned centrally. Eight over-demanded exam schools or classes still had decentralized admissions similarly to the procedure already described in 2011. However, there have been some minor modifications. All the examinations are run on the same weekend to force families to represent only few preferences, which can of course become a more strategic choice than it used to be. Students have to consider the competitive nature of the school market and in their to strategic revaluation of preferences, they have to consider which school they are more likely to be accepted to. From the other side of the coin, schools seek for more efficient admission procedure by constricting preference lists by time to shorten the application procedure and thus find a more efficient match for the proposing side.

In addition, the application procedure to regular schools has been changed. Firstly, regular school applications have been limited to three options, i.e. the parent has the right to list three schools, but these are not considered as an ordered list. Secondly, the application can contain information about siblings and their school. Centralized school priorities are considered as follows: (1) siblings, and (2) distance from school (in meters) from the officially registered
address. We argue that in regular schools, a centralized matching common utility SD was used, by the latter, we mean the sort of school selecting one-at-a-time matching, where the schools are considered in a random order. Thus, we define 2012 onwards Tallinn mechanism as a hybrid:

Step 1 Students are assigned to exam schools based on decentralized schools proposing DA (ScDA) algorithm (described in previous section);

Step 2 Remaining students are centrally assigned to regular schools based on RSD mechanism using priority lists drawn up by local authority (ScRSD).

a Create randomly ordered list of schools, with exam schools always being the first.

b Iterate over all the schools in the random order. In each school assign students to that school with siblings. If there are more students than seats use distance to prioritise (unlikely to happen).

c Again iterate over all the schools in the random order. In each school assign students to that school with highest priority (shortest distance) until quota is full.

School priorities are set by the principle of 'common utility'. The following priority classes are used in Tallinn: (a) siblings in the same school; (b) distance from home (in kilometres). And the procedure that follows is school random serial-dictatorship, as each school can select his highest priority students and schools are considered in a random order.

The admission process to the exam schools takes place between January and March. It has been shifting from March (in 2012) to February (in 2013) and even to January (in 2014). The second stage (step 2) in regular schools starts on 1 March with the submission of an electronic application to the e-school register. Central but manual entries are made by 25 May. By 10 June, parents must either accept or decline offers. There is late and decentralized round of applications open after 15. of June.

3 Examples of matchings

In this section, we develop our ideas via several examples. First example illustrates the functioning of the Tallinn mechanism open enrolment or 2011 mechanism. It concerns school-proposing matching in Tallinn exam schools and assumes the process has enough time to execute. We represent students’ preferences as a linear order of schools to which he prefers to be assigned (assuming truthful revelation of preferences).

Example 1. There are four students \{a, b, c, d\}, and four schools \{s_1, s_2, s_3, s_4\}. Students preferences and school priorities are given as follows:

\[
\begin{align*}
\text{a} & : s_1 - s_2 - s_3 - s_4 \\
\text{b} & : s_2 - s_3 - s_4 - s_1 \\
\text{c} & : s_3 - s_4 - s_1 - s_2 \\
\text{d} & : s_4 - s_1 - s_2 - s_3 \\
\text{s_1} & : b - c - d - a \\
\text{s_2} & : c - d - a - b \\
\text{s_3} & : d - a - b - c \\
\text{s_4} & : a - b - c - d
\end{align*}
\]

Matching with school proposing DA (ScDA) gives us following result:

\[
\mu_{\text{ScDA}} = \begin{pmatrix}
 a & b & c & d \\
 s_4 & s_1 & s_2 & s_3
\end{pmatrix}
\]

This results is granting last preference for all students. Whereas if students start the proposing procedure instead of schools the result will be:
\[ \mu_{StDA} = \begin{pmatrix} a & b & c & d \\ s_1 & s_2 & s_3 & s_4 \end{pmatrix} \]

It is also shown by Roth and Sotomayor (1990) that the set of possible stable matching is a lattice (partially ordered set), where on the one end there is a school-optimal matching and on the other student-optimal matching.

Second example illustrates the trade-off between stability and efficiency.

**Example 2.** There are three students \{a, b, c\}, and three schools \{s_1, s_2, s_3\}. Students preferences and school priorities are given as follows:

- \( a : s_2 - s_1 - s_3 \)
- \( s_1 : a - b - c \)
- \( b : s_1 - s_2 - s_3 \)
- \( s_2 : c - a - b \)
- \( c : s_1 - s_2 - s_3 \)
- \( s_3 : a - b - c \)

The DA student-optimal matching is:

\[ \mu_{StDA} = \begin{pmatrix} a & b & c \\ s_1 & s_3 & s_2 \end{pmatrix} \]

While in latter both \( s_1 \) and \( s_3 \) get their second preferred schools. Thus both of them will be better-off if mechanism will allow trade:

\[ \mu_{TTC} = \begin{pmatrix} a & b & c \\ s_2 & s_3 & s_1 \end{pmatrix} \]

Latter is a Pareto improvement for students compared to StDA for the students \( a \) and \( c \). However, the new allocation includes a blocking pair – \( b \) prefers \( s_1 \) and \( s_1 \) also prefers \( b \) to the current match \( c \). The example indicates that even in the case of SOSM we can easily show that Pareto efficiency is not granted, and alternative mechanisms (TTC) can allow welfare improvements with stability trade-offs.

Second example describes the Tallinn mechanism after 2011. We show how various components will affect matching outcomes, e.g. limiting number of preferences, not considering linear order of preferences, not running the DA until it terminates. To continue with the example over the properties of Tallinn mechanism when we have a deadline to stop the process. For simplicity we terminate the algorithm after the first step, i.e. all students accept their first proposal.

**Example 3.** Same preferences as in example 2: there are three students \{a, b, c\}, and three schools \{s_1, s_2, s_3\}. Students preferences and school priorities are given as follows:

- \( a : s_2 - s_1 - s_3 \)
- \( s_1 : a - b - c \)
- \( b : s_1 - s_2 - s_3 \)
- \( s_2 : c - a - b \)
- \( c : s_1 - s_2 - s_3 \)
- \( s_3 : a - b - c \)

Assume that ScDA is executed and terminated after first round. If a student receives more than one proposal, he will accept her first preference. School proposing DA will result in:

\[ \mu_{ScDA'} = \begin{pmatrix} a & b & c \\ s_1 & s_2 & s_3 \end{pmatrix} \]

We see that the matching is neither efficient nor stable. For example, student \( a \) would prefer \( s_2 \), while also \( s_2 \) would prefer \( a \) to \( b \). Contrary to TTC now two students are allocated to their second-best choice while third student is getting his least preferred school.

**Example 4.** Same preferences as in example 2: there are three students \{a, b, c\}, and three schools \{s_1, s_2, s_3\}. Students preferences and school priorities are given as follows:
Table 2: Cases for evaluating presented preferences

<table>
<thead>
<tr>
<th></th>
<th>Full knowledge</th>
<th>Ignorance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strict preferences</td>
<td>Case 1</td>
<td>Case 3</td>
</tr>
<tr>
<td>Relatively indifferent preferences</td>
<td>Case 2</td>
<td>Case 4</td>
</tr>
</tbody>
</table>

\[
a: s_2 - s_1 - s_3 \quad \quad s_1 : a - b - c \\
b: s_1 - s_2 - s_3 \quad \text{and} \quad s_2 : c - a - b \\
c: s_1 - s_2 - s_3 \quad \quad s_3 : a - b - c
\]

Assume that each student can present only two preferences which are not ordinarily listed. The exercise has now two components: (1) will student reveal true preferences; (2) what will be the outcome of school RSD? To answer the first part we consider now two aspects. Firstly, student needs additional information to evaluate the demand at each school for strategic choice. Second, instead ordinal preferences cardinality (strictness of the preference order and relative utility related with certain schools) of preference revaluation matters. Hypothetically we arrive at four potential cases as in Table 2.

**Case 1** The preferences are strict. The student knows the demand, but the do not necessarily knows what others report under strategic manipulation with revealed information. Assuming that everybody reports truthfully their first two preferences. If we take the order of dictatorship to be \{s_1, s_2, s_3\}, the resulting matching would be

\[
\mu = \begin{pmatrix} a & b & c \\ s_1 & s_2 & \end{pmatrix}
\]

It can be seen that become assigned student \(b\) has to evaluate his priorities at lower ranked school and would be better off if instead \{s_1, s_2\} he would report \{s_1, s_3\}, the result for \(b\) would be \(\mu(b) = s_3\). Moreover, it can be rational for the students \(a\) and \(b\) to limit their preference lists. By assuming some strategic behaviour the reported preferences could be:

\[
a: s_2 \quad b: s_1 - s_3 \quad c: s_1
\]

ScRSC will allocate seats:

\[
\mu = \begin{pmatrix} a & b & c \\ s_2 & s_1 & \end{pmatrix}
\]

Here again \(c\) would rather report \(s_3\). So we have to analyse the equilibrium solution of the reporting game. Each students has 6 strategies for reporting one or two schools. This results in altogether \(6^3 = 216\) pure strategy solutions. We will look at the structure of equilibrium solutions. From the preferences we see that all students prefer to be matched than unmatched and there are enough seats in schools for all students. So in every equilibrium solutions all students have to be matched. There is a multitude of pure strategy equilibria, but all result in the same allocation \(\mu(a, b, c) = (s_1, s_3, s_2)\). There are 21 pure strategy equilibrium strategies: \(a = \{s_1, \{\}\}, b = \{s_3, \{\}\}, c = \{s_2, \{\emptyset, \{s_3\}\}\}\) and \(a = \{s_1, \{\}\}, b = \{s_1, s_3\}, c = \{s_2, \{\{s_1\}, \{s_3\}\}\}\). If \(b = \{s_3\}\) and \(c = \{s_2, s_1\}\) then \(a\) would rather report \(s_2\) than \(s_1\). With \(\{\}\) being shorthand for anything else not reported yet, i.e. the \(\emptyset\) or some other school.
With alternative orderings of schools the allocation stays the same and the equilibria are \( a = \{s_1, \emptyset, \{s_3\}\}, \ b = \{s_3, \cdot\} \) and \( c = \{s_2, \cdot\} \); \( a = \{s_1, \emptyset, \{s_3\}\}, \ b = \{s_1, s_3\} \) and \( c = \{s_2, \cdot\} \); \( a = \{s_1, \emptyset, \{s_3\}\}, \ b = \{s_3, \cdot\} \) and \( c = \{s_2\} \). In the latter case there are only 6 equilibria.

Obviously we cannot assume that the players can comprehend the whole game. Already in this example there are 216 strategies to be evaluated. But the overall observation is that in most cases it makes sense to report just one school and not your first preference.

Interestingly enough the match is identical to the student-optimal matching with deferred acceptance, but its main drawback is the strategical complexity, indicating that students don’t have dominant strategy.

**Case 2** We assume that students have weakly preferences over schools. Full information assumption is hold, thus if some schools are more preferred by others it is rational not to list them.

**Case 3** We assume that there is no pre-knowledge of any kind about the demand over schools. Even in the case of strict preferences it is unclear what students report. There can be many strategies, i.e. report at least second preference and something else. The case needs further investigation.

**Case 4** We assume no knowledge about demand over the schools and weak preferences. In this case the strategy is obvious and simple. As students do not strictly prefer one school to another they should report two random schools from their list.

### 4 Properties of the Tallinn matching mechanism

In this section, we use data from the e-kool electronic register. Each year approximately 4000 students start school in Tallinn. There are around 60 primary schools. We investigate the characteristics of the Tallinn 2011 (market experiment) and hybrid mechanism (after 2011); and compare these to DA, TTC and Boston.

#### 4.1 Discussion of the hybrid mechanism

One primary concern is that the current design and implementation of the Tallinn hybrid mechanism and Tallinn 2011 will hurt participants, mainly students, the very people it is intended to help.

**Observation 1:** Tallinn hybrid mechanism for the exam schools School proposing DA (ScDA) with early stop may not be student-optimal nor stable.

Roth (2008) demonstrates that school proposing DA need not be strategy-proof for the schools or students, intuitively since schools have multiple places they have more options for manipulation. From the students’ perspective this could also be considered as group strategy-proofness, where a group of students cannot manipulate their preferences for a better outcome.

Excluding discussion about the unacceptability of the allocation of primary school places using test scores or some other ability measures, we observe that most of the current problems related to the exam school mechanism are inherited from the decentralized nature of the process. As the decentralized acceptance process has a time limit, it may force students or schools to close the allocation before finding an optimal match.

**Observation 2:** If student preferences are not considered in linear order by the mechanism and in the case of varying cardinal utility over schools including full knowledge over demand
then the students are motivated to list only single school. In the case of weak preferences it is rational to list more than one alternative.

4.2 Empirical results: preference revelation

Observation 2 is supported by the empirical evidence indicated in the Table 3. From Table 3 we see that in 2011 students presented more preferences than in following years 2012 and 2013. This would be an obvious conclusion from our Example 4., where in case on ScRSD students have an incentive to present less preferences. Thus Tallinn mechanism is forcing students into manipulations, i.e. truthful revelation of preferences is not a rational strategy.

<table>
<thead>
<tr>
<th># of prefs</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>52 %</td>
<td>74 %</td>
<td>76 %</td>
</tr>
<tr>
<td>2</td>
<td>18 %</td>
<td>15 %</td>
<td>14 %</td>
</tr>
<tr>
<td>3</td>
<td>11 %</td>
<td>11 %</td>
<td>9 %</td>
</tr>
<tr>
<td>&gt;3</td>
<td>18 %</td>
<td>0 %</td>
<td>0 %</td>
</tr>
<tr>
<td>Mean</td>
<td>2.2</td>
<td>1.4</td>
<td>1.3</td>
</tr>
</tbody>
</table>

4.3 Empirical results: comparison of the allocations

We conduct an empirical study of the matching mechanisms in tree consecutive years 2011, 2012 and 2013. First, 2011 was the year that Tallinn officials gave up any kind of zoning or other distance-based priorities, and decentralized the market. This experiment lasted only for a single year and was transformed to a semi-centralized (hybrid) market in 2012. In 2013 there were no major changes of the mechanism.

In 2011 there were 4346 students in total, in 2012 there were 3028 and in 2013 3422 students. The sudden decrease is resulting from the change of admission principles. i.e. under hybrid decentralised first round is not registered in e-kool data (approximately 1000 students). In the proposing side, there are 64 schools, 10 of them are exam schools (matched during the first round in 2012 and 2013) and the rest are regular schools. At the same time in public schools no all places were finally allocated (unassigned), indicating that relatively large number of students must have some sort of outside option private schooling, a school in another municipality, home schooling etc.

There are some simplifying assumptions made compared to the actual mechanisms. As our examples and observations allow to conclude the students do not report their actual preferences. Thus we randomly generate 100 different strict preference orderings over the presented set. Obviously these will not reflect the actual preferences of the students and assumes some naive behaviour, but give us some comparative data. In most cases we compare allocations of five matching mechanisms:

Tallinn the actual allocation;

ScRSD school proposing random serial dictatorship (Step 2 in the hybrid Tallinn mechanism) using generated preferences;

StDA student proposing deferred-acceptance using generated preferences;
**TTC** Top-trading cycles using generated preferences;

**Boston** Boston mechanism using generated preferences and assuming truthful reporting.

We report results with several metrics: (a) Welfare of the society/schools, measured by average distance between schools and student’s home (Table 4); (b) Percentage of students assigned to the same schools with their siblings (Table 7); (c) percentage of unassigned students (Table 5); welfare of students, measured by percentage of students who received their first preference (Table 6). All reported differences are statistically significant with on level $P(|t|) < 0.01$ in pairwise two-sided t-test.

### Table 4: Welfare of the society: Priority utilisation

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Mean distance (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2011</td>
</tr>
<tr>
<td>Tallinn</td>
<td>2.03</td>
</tr>
<tr>
<td>ScRSD</td>
<td>1.57</td>
</tr>
<tr>
<td>StDA</td>
<td>1.77</td>
</tr>
<tr>
<td>TTC</td>
<td>1.85</td>
</tr>
<tr>
<td>Boston</td>
<td>1.88</td>
</tr>
</tbody>
</table>

Tallinn mechanism priorities students by how far they live from the school. Thus, technically we may say that political aim is to minimise the distance that students need to travel to school. In Table 4 comparison are shown on yearly bases. Overall we see that by applying an algorithmic approach we do not perform more that 10% worse than Tallinn. However, considering the number of unassigned students in Tallinn it is huge (Table 5), 16% to 18% on average, when compared to any mechanism it is at least three times worse. A limitation to current result can be the diligence in updating the status of student’s assignment is poor and the matching data don’t reflect all matched (and accepted) pairs.

In addition, it is visible that the manual and decentralized DA (Tallinn 2011) is under-performing from societal perspective in minimizing distance. By giving more consideration to student preferences, we only slightly loose on overall average distance, while assigning many more students according their first preference by StDA. This is an evidence that the decentralized market suffers from timing problems or/and human errors.

### Table 5: Welfare of the students: Unassigned students

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>% of unassigned</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2011</td>
</tr>
<tr>
<td>Tallinn</td>
<td>16%</td>
</tr>
<tr>
<td>ScRSD</td>
<td>6%</td>
</tr>
<tr>
<td>StDA</td>
<td>3%</td>
</tr>
<tr>
<td>TTC</td>
<td>4%</td>
</tr>
<tr>
<td>Boston</td>
<td>3%</td>
</tr>
</tbody>
</table>

We cannot include preference utilisation in the case of actual Tallinn mechanism into comparison, because the ordered preferences are not reported. Thus our second-best empirical
strategy is to compare the well-known algorithms with the hypothetical ScRSD. Here we see that the result is not considerably worst when comparing unassigned students or average distance. When we start comparing the resulting preferences in a matchings we see significant differences (Table 6). With the ScRSD about 10% less students would receive their first preference and result is even worse when preference lists are longer as in year 2011, when only 68% would had received their first preference.

Table 6: Welfare of the students: Received first preference

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>% of first preference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2011</td>
</tr>
<tr>
<td>ScRSD</td>
<td>68%</td>
</tr>
<tr>
<td>StDA</td>
<td>92%</td>
</tr>
<tr>
<td>TTC</td>
<td>93%</td>
</tr>
<tr>
<td>Boston</td>
<td>95%</td>
</tr>
</tbody>
</table>

In general the number of students who received their first preference is susceptible to manipulation. As we saw from the example students might not want to report their true preferences. While this is certainly true we are unsure how many students or families actually are able to compute the best strategy. In general, as literature shows the truthful preference revelation has merits on his own, i.e. not forcing parents to find the strategy for the optimal manipulation. In aggregate (Table 6) we see that with ScRSD mechanism we would find a suboptimal allocation, thus suggesting that there is room for manipulation.

Table 7: Welfare of the students: Siblings

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>% from total students with siblings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2011</td>
</tr>
<tr>
<td>Tallinn</td>
<td>NA</td>
</tr>
<tr>
<td>ScRSD</td>
<td>NA</td>
</tr>
<tr>
<td>StDA</td>
<td>NA</td>
</tr>
<tr>
<td>TTC</td>
<td>NA</td>
</tr>
<tr>
<td>Boston</td>
<td>NA</td>
</tr>
</tbody>
</table>

Regarding siblings we see that ScRSD (Table 7) is doing really well. Compared to Tallinn actual mechanism we may conclude that manual execution of the mechanism we do intentional or unintentional errors. Other mechanisms (StDA, TTC and Boston) are not that much different from the ScRSD.

5 Conclusions

We deconstructed the current Tallinn mechanism being used for primary school place allocations and found it to be a hybrid of the decentralized school proposing deferred acceptance algorithm for inter-district exam schools and a centralized common utility serial dictatorship for the intra-district regular schools. The latter is a mechanism where students are randomly listed and matched many-to-few, according to school priorities which we have named common utility. There are currently two priority groups (without quotas): siblings and distance from the school.
To the best of our knowledge, this hybrid is not an often-referenced mechanism in school choice literature, i.e. Abdulkadiroglu (2011) describes the high school mechanisms in Boston Public Schools as a relatively similar, albeit student proposing, procedure. Similar to our Step 1 in the hybrid, Roth (2008) has described decentralized matching markets in other situations. One example of the hybrid mechanism (Boston or first-preferences-first) have been outlawed in the United Kingdom mainly because applicants must misrepresent their preferences in order to get into a preferred school, but not all manage to determine what the optimal manipulation might be Pathak and Sönmez (2011). We see some similar tendencies in the case of Tallinn. First, decentralized entrance exams limit the number of preferences submitted and make students act strategically, that is, not choose the most competitive schools, even if they are at the top of their preference list. Second, the decentralized nature of the market does not absorb all the benefits from deferred acceptance, because of the time limit. We argue that neither of the mechanisms in the hybrid are without the problems of instability and inefficiency. However, school proposing deferred acceptance need not harm the equal treatment (stability) property of the mechanism. In addition to the possibility of harming student welfare, the problem arises from the decentralized and time-limited execution of the algorithm that has also been observed by Roth (2008) for some markets in US.

On the results we saw that the actual matching in Tallinn is biased towards minimising distance and taking less account students preferences and siblings. If the matching has been centrally without manual execution by school proposing random serial dictatorship then the social and individual utility could be improved simultaneously (excluding some trade-off in average distance).

Overall we see that by using an student-proposing deferred-acceptance mechanism in Tallinn we would obtain a superior solution for students, while not losing much in societal preferences, e.g. distance from school. Also it would be rational for parents to report truthful preferences, then they can concentrate on forming their preferences rather than finding a way of reporting manipulated preferences.

References


