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Wind-driven ocean upper layer model
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Ekman layer; modelling; turbulence.

EXTENDED ABSTRACT

The observed current dynamics in the wind-driven upper layer of the ocean deviates from the classical Ekman theory. So, the surface current veering from the wind stress direction differs in general from the value predicted by the Ekman theory and the currents' turn (clockwise in the northern hemisphere) with depth is often smaller than in the Ekman theory (Price et al., 1987). The deviations have been so far explained by complementing the Ekman theory with either the logarithmic boundary layer model, with the surface-wave-induced velocity bias, with the depth-varying eddy viscosity or with the ocean density stratification (Price and Sundermeyer, 1999). Our hypothesis is that the observed deviations of the surface current from the Ekman theory can be explained by the prevailing orientation of large-scale eddy rotation in the ocean upper layer supported by the velocity shear. The suggested model is based on the relevant turbulence theory (Heinloo, 2004) treating the turbulence as divided into the orientated (large-scale) and non-orientated (small-scale) constituents. The dominant effect is attributed to the orientated turbulence constituent interacting immediately with the average flow.

Let \((x,y,z)\) be the wind-relative Cartesian coordinate system with the coordinate \(z\) directed downward, \(z=0\) at the ocean surface, and the coordinate \(x\) directed down the wind. The flow is considered forced by the constant wind stress \(\tau = (-\tau,0,0)\), \(\tau > 0\). The coordinate system is assumed right-hand in the Northern Hemisphere and left-hand in the Southern Hemisphere. Assuming the flow velocity \(\tilde{u} = u_x + iu_y\), where \(i\) is the imaginary unit, depending on \(z\) only, the solution of equations of the applied theory for the specified flow in the non-stratified ocean has the form

\[
\tilde{u} = \tilde{u}_1(0)\exp(-\frac{z}{h_{11}})\exp(-i\frac{z}{h_{12}}) + \tilde{u}_2(0)\exp(-\frac{z}{h_{21}})\exp(i\frac{z}{h_{22}}),
\]

in which

\[
\tilde{u}_1(0) = \frac{\tau}{\mu_s} \frac{-\lambda_1 + B\lambda_2 \exp(i\pi/4)}{\lambda_1^2 - \lambda_2^2} \quad \text{and} \quad \tilde{u}_2(0) = \frac{\tau}{\mu_s} \frac{\lambda_2 - B\lambda_1 \exp(i\pi/4)}{\lambda_1^2 - \lambda_2^2}.
\]

In (1) and (2): \(\lambda_1 = -h_{11}^{-1} - ih_{12}^{-1}\) and \(\lambda_2 = -h_{21}^{-1} + ih_{22}^{-1}\), where the depth scales \(h_{11}, h_{12}, h_{21}\) and \(h_{22}\) are determined by the coefficients characterizing the medium physical properties (Heinloo 2004), while \(h_{21} > h_{11} > 0, h_{21} > h_{22} > 0, h_{12} > h_{11} > 0\) and \(h_{12}h_{21} + h_{11}h_{22} = h_{12}h_{21} - h_{11}h_{22}\).
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\( B > 0 \) and \( \mu_s > 0 \) are determined by the medium physical parameters and by the boundary condition of the average angular velocity of the eddy rotation at the ocean surface. The solution (1) includes the observed deviations of the velocity vertical distribution from the velocity predicted by the classical Ekman solution, following from (1) for \( \tilde{u}_1(0) = 0, \ h_{21} = h_{22} = \ell_E \), where \( \ell_E \) is the Ekman depth scale, and \( \tilde{u}_2(0) = |\tilde{u}(0)| \exp(i\pi/4) \).

In Fig.1 the solution (1) is compared with data on the flow velocity relative to the reference level at \( z = 98 \) m measured in the Drake Passage, Southern Ocean adopted from (Lenn and Chereskin, 2009) and with the corresponding velocity distributions according to the Ekman solution. It can be seen in Fig.1 that the depth of the Ekman layer following from the suggested model and from the Ekman solution exceeds considerably the depth of the applied reference level of 98 m resulting in different estimates of the velocity e-folding depth. In particular, the stated in (Lenn and Chereskin, 2009) difference of 2-3 times between the velocity e-folding scale and the rotation scale (see also (Price et al., 1987; Schudlich and Price, 1998; Price and Sundermeyer, 1999)) would not show up for the reference depth compatible with the theoretical vertical distribution of velocity.

**Figure 1.** The depth-dependence of velocity components \( \Delta u_x = \text{Re} \Delta \tilde{u} \) and \( \Delta u_y = \text{Im} \Delta \tilde{u} \), where \( \Delta \tilde{u} = \tilde{u} - \tilde{U} \) in which \( \tilde{U} \) is the velocity at the reference level \( z = 98 \) m, calculated according to the suggested model for \( B = 1.7, \ h_{11} = 14.0 \) m, \( h_{21} = 44.2 \) m, \( h_{22} = 43.6 \) m, \( \tau \mu_s^{-1} = 0.115 \) s\(^{-1}\) (solid curves) and according to the Ekman model for the Ekman scale \( h_E = 44 \) m, \( |\tilde{u}(0)| = 3.2 \) cm s\(^{-1}\) (dashed curves) compared with data (circles) adopted from (Lenn and Chereskin 2009).

**REFERENCES**


