Identification of areas of frequent patch formation from velocity fields

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ABSTRACT


We explore the ability of the formation of patches of substances floating on the sea surface owing to intrinsic features of ocean circulation associated with persistent flow convergence in areas characterised with strong vertical velocities (e.g. hosting downwelling). Their impact on the field of surface floaters can be quantified using so-called flow compressibility of a two-dimensional velocity field. Large values of this measure for idealized, delta-correlated-in-time flows are directly related to the tendency of floating tracers to gather into patches. We employ a modification of this measure, so called finite-time-compressibility to describe the real marine flows in a more consistent way, accounting for the match of areas with high convergence with the Lagrangian transport of the resulting patches. Analysis of this measure for the Gulf of Finland, the Baltic Sea, using surface velocity fields from the OAAS model with a resolution of 1 km shows that domains where systematic development of patchiness is very likely occur either along straight sections of the coastlines that usually host downwelling. Surprisingly, high values of finite-time compressibility frequently occur throughout the year in two offshore locations and in the windy season near the centre of the widest part of the gulf.

ADDITIONAL INDEX WORDS: flow compressibility, finite-time compressibility, patchiness.

INTRODUCTION

The probability of different coastal sections of being hit by high concentrations of adverse impacts (algal blooms, marine litter, oil pollution, etc.) is closely related to both the potential of the formation of patches of substances floating on the sea surface and the subsequent transport of these patches to the nearshore. The process of patchiness formation (Powell and Okubo, 1994), although not particularly well understood, reflects the ability of substances and tracers in the surface layer to naturally form areas with high concentrations. This process has been thoroughly examined for the nearshore areas and for semi-sheltered sea domains (e.g., Kononen et al., 1992; Granskog et al., 2005) where it is frequently driven either by local bathymetry, coastal jets and/or associated intense up- or downwelling events, or by anthropogenic intervention (dredging, dumping, etc.). It is frequently coupled with the generation of strong gradients in some domains and of a mixture of areas with highly different concentration of various substances (Kalda, 2000), which is another natural ocean process, basically driven by the inverse energy cascade of the almost two-dimensional (2D) large-scale motions (Cushman-Roisin and Beckers, 2011); however, this process only redistributes areas with high concentration of different substances and does not amplify the concentration.

There is increasing evidence that certain parts of open ocean that are located far from strong jets (e.g., so-called Great Pacific Ocean Garbage Patch (Pichel et al., 2007) located in the North Pacific Subtropical Convergence Zone where marine litter has a tendency to gather) and several nearshore areas governed by essentially three-dimensional (3D) motions systematically develop patches of high concentrations of certain substances. The formation of such areas is usually associated with large-scale convergence and subduction zones (Lee and Niiler, 2010) characterised by strong vertical velocities. Similar properties may have nearshore areas hosting up- or downwelling, or areas of interaction of jet-like currents and mesoscale vortices.

The potential impact of the velocity patterns towards the formation of patches of high concentration in the initially more-or-less homogeneous fields of surface floaters or tracers can be quantified using so-called flow compressibility $C_f$ of a 2D velocity field on the sea surface, which lies over the 3D circulation. Differently from the compressibility of the medium, the flow compressibility reflects the divergence or convergence of the flow field. It is defined as the relative weight of the irrotational (curl-free or potential) component of the flow (which is generally composed of potential and solenoidal components). Large values of this measure are directly related to the tendency of floating tracers to gather into patches (Cressman et al., 2004; Kalda, 2007) because of the capability of water particles to “dive” (Garrison, 2011) whereas the floating particles are locked at the surface and their concentration therefore increases. While spatio-temporal variations in the divergence field generally lead to a certain domination of the gathering of the floaters (see discussion of this effect in Giudici et al., 2012; Kalda et al., 2012), the areas hosting high values of $C_f$ are inherent candidates for domains where the natural formation of patches of dangerous concentrations of pollution may occur. The presence of such areas could potentially affect both the probability and propagation time of adverse impacts from the offshore areas to the vulnerable regions (Chrastansky and Callies, 2009; Soomere et al., 2010). Systematic formation of areas with high concentration of tracers only happens for relatively high values flow compressibility. In
ideal Kraichnan flows (which are delta-correlated in time), the clusterization starts only if $C_c > 0.5$ (Falkovich et al., 2001). Although for smaller values of $C_c$, theoretical studies predict that there are no distinct patches of floaters but still there are regions of increased concentrations (Bec et al., 2004; Boffetta et al., 2004). It has been also shown that if the floater particles stick to each other (e.g. like pieces of plastic sheets do), the patch formation starts at considerably smaller values of flow compressibility (Kalda, 2007).

The presence of high values of $C_c$ not necessarily leads to the increase in the concentration of particles. Samuelsen et al. (2012) demonstrated that particle aggregation systematically occurred in the rim of an anticyclonic eddy. The highest particle concentration coincided with the vorticity patterns, but not the divergence field. The spatial de-correlation between the divergence field and the areas of high concentration apparently occurred because the areas of strong convergence stayed fixed in space relative to the eddy, while the particles were advected with the currents. In other words, while areas of high convergence may impact the concentration field, the largest concentration anomalies occur if a set of particles stays in such areas for a longer time interval. Such areas are the most natural candidates for patch formation even if the instantaneous values of divergence remain relatively modest.

The classical definition of flow compressibility relies on certain instantaneous properties of the flow field and ignores its history. Both experimental and numerical results indicate that time correlations (which are always present for real hydrodynamic flows) can either inhibit or catalyze the clusterization process (Boffetta et al., 2004). For this reason, Giudici et al. (2012) and Kalda et al. (2012) employed a modified measure called finite-time compressibility. This measure is not only directly related to the ability of some regions of the sea surface to shrink or expand but also accounts for finite-time correlations (the history) of the motion, and thus has a larger potential to describe the formation of patches. It essentially relates the finite-time changes in the material volumes (e.g. surface areas in 2D flows overlying 3D circulation) to the finite-time changes in the separation between material particles.

Previous research (Giudici et al., 2012; Kalda et al., 2012) has established the basic relations between the classical flow compressibility and finite-time compressibility, and also demonstrated the link between the finite-time compressibility and divergence field. In this paper we focus on long-term spatial distributions of the finite-time compressibility in a particular sea domain that is famous for the high proportion of vertical motions of water masses—the Gulf of Finland, the easternmost sub-basin of the Baltic Sea (Leppäranta and Myrberg, 2009). This sea is characterized by frequent occurrence of 3D dynamics (Myrberg and Andrejev, 2003). The Gulf of Finland frequently hosts powerful up- and downwelling events (Lehmann and Myrberg, 2007) and thus is a natural example of a domain with substantial nonzero values of flow compressibility in the surface layer. The focus is on the identification of areas where the finite-time compressibility exceeds the threshold for the clusterization in terms of the classical flow compressibility.

**METHOD AND DATA**

Finite-time compressibility

As we are basically interested in changes in the concentration of substances or particles passively carried by the flow (below called simply (floating) particles) in the surface layer, we focus on the calculation of relative changes in the surface elements that are passively carried with the surface current. We track the displacement of and changes to triangles formed by a selected point of the sea surface and its immediate neighbours to the north and east (Figure 1) to represent the change in the surface area at this point. This stencil is applied to the entire sea domain of interest. The values of the finite-time compressibility $C_{ftc}$ are calculated from the root-mean-square (rms) changes to such triangles (equivalently, from the separation between triplets of particles floating on the surface) over a certain time interval (Kalda et al., 2012):

$$C_{ftc} = \frac{2dS_{rms}}{dA_{rms} + dB_{rms}}.$$  (1)

Here $S_i$ is the surface of such an initially right-angled triangular element at a time instant $t$, $dS_i = \frac{1}{2}(S_i - S_{i+1})$, $S_i$ is the relative change of its area over one time step, and $dA_i = (A_i^2 - A_i^{2,1})/A_i^2$, $dB_i = (B_i^2 - B_i^{2,1})/B_i^2$ are changes to the squared length $A_i^2$, $B_i^2$ of its catheti. By definition, this measure characterizes persistent changes in the surface of such elements that are carried with surface current and thus the "ability" of a tracer, substance or concentration field to develop extensive variations (patches) in some sea areas. For a purely incompressible flow $dS_{rms}$ and hence $C_{ftc} = 0$. If the flow is purely compressible (contractive or expanding), the quantities $A$ or $B$ would change with the same relative rate as $S$ and $C_{ftc} = 1$. The values of $C_{ftc}$ are expected to mostly lie in the range $[0,1]$.

The classical flow compressibility $C_c$ and the quantity $C_{ftc}$ are highly correlated but not equivalent (Giudici et al., 2012). For delta-correlated, ideal Kraichnan flows the relation between the two quantities is:

$$C_{ftc}^2 = \frac{2C_c}{2C_c + 1}.$$  (2)

The two measures simultaneously tend to zero. The threshold $C_c = 0.5$ for the process of clustering (Falkovich et al., 2001) corresponds to $C_{ftc} = \sqrt{1/2} \approx 0.7$. For moderate levels of compressibility $C_{ftc} > C_c$. The two measures only coincide if $C_{ftc} = C_c = 0.781$. For larger values of $C_c$, the $C_{ftc}$ exceeds the flow compressibility. The limiting value $C_c = 1$ corresponds to $C_{ftc} = \sqrt{2/3}$. This implies that for $\sqrt{2/3} < C_{ftc} \leq 1$, there are no corresponding values for $C_c$ (Giudici et al., 2012).

**Velocity data for the Gulf of Finland**

Since the flow compressibility values of a velocity field coincide with those of the motion of fluid particles floating in it, we study the compressibility of a velocity field through the

![Figure 1. Elementary triangular set of floating particles used for the calculations.](image-url)
displacement vector fields of simulated particles’ trajectories generated by that velocity field. The previous studies in this test area (Giudici et al., 2012; Kalda et al., 2012) relied on velocity fields with a moderate spatial resolution of 2 nautical miles (nm), which only to some extent resolved the basic features of dynamics in this water body. In this study we use velocity fields with a resolution of 1 nm. This means a considerable improvement of the adequacy of the results as the 1 nm models are expected to properly resolve most of the mesoscale dynamics in the Gulf of Finland.

More specifically, 2D surface velocity fields were extracted from long-term numerical simulations using the OAAS hydrodynamic model (Andreyev and Sokolov, 1989; 1990) and performed in the framework of BONUS+. BalticWay cooperation. This time-dependent, free-surface, baroclinic model based on hydrostatic approximation has a vertical resolution of 1 m (and only the uppermost layer is assumed to be 2 m thick). It was specifically designed for areas characterized by complex hydrostatic and bathymetric structures, such as the Gulf of Finland. Its latest features and characteristics are presented in Andreyev et al. (2010). The particular model run (Andreyev et al., 2011) was forced with meteorological data from a regionalization of the ERA-40 re-analysis over Europe using a regional atmosphere model with a horizontal resolution of 25 km (Högland et al., 2009; Samuelsson et al., 2011). The velocity field covers the Gulf of Finland to the west of longitude 23°27′E during years 1987–1991, with a temporal resolution of 3 hours. As in the previous studies, we only used the velocity data in the uppermost layer.

Set-up of numerical experiments

We developed a simulation environment (QTRAC) to calculate spatial distributions of $C_{\text{pt}}$ from velocity fields. As the first step of the calculation pipeline, QTRAC loads the vector 2D velocity information and stores it in a staggered Arakawa B-grid (Arakawa and Lamb, 1977). An (offline) interpolation in time is then optionally performed to fit the data with any resolution chosen for the simulations. The velocity is assumed to vanish within dry grid cells and outside the boundaries of the model.

The scheme implied in the evaluation of the finite-time compressibility of semi-persistent surface flow patterns (Soomere et al., 2010). The time interval $t_{\text{obs}}$ of interest (normally from a month up to first years) was divided into shorter (optionally overlapping) time spans of custom length (typically 4–96 hours in various experiments, 24 hours in the simulations used in this study). One particle was positioned at the centre of each wet grid cell of the model. The particles were thus separated by exactly 1 nm from their closest neighbours.

The displacement of all released particles was then tracked throughout the chosen time span using the assumption that the particles are passively carried by the modelled velocity field. No means to replicate the effect of subgrid-scale motions is used. The coordinates of the resulting set of trajectories are sampled at a constant rate (30 min in this paper). The position of the particles was then reset and a new set of trajectories was calculated over the next time span that started one day (24 hours) after the previous one.

The trajectories of floating particles are resolved through an Euler-type Runge-Kutta procedure, a first-order method that yields a global error which is proportional to its time step size (Kloeden and Platen, 1999). Whilst higher-order schemes would be definitely necessary to properly resolve the vertical component of the trajectories in strongly stratified environments such as the Baltic Sea (Gräwe et al., 2012), we limit our trajectories in the uppermost layer where the Euler scheme is commonly believed to give proper results.

We use the local velocity vector calculated via double linear interpolation from the modelled velocity components at four closest grid points to the instantaneous location of the trajectory. The next location of the particle along its trajectory is calculated component-wise. After each simulation over the chosen time span has completed, each batch of trajectories is stored. The trajectory of each floating particle is then linked with its two closest neighbours at the initial time step according to the stencil in Figure 1. This results in a set of triangular elements.

A value of the finite-time compressibility associated with each grid cell and the initial time of each time span is then evaluated using Eq. (1), the values of the catheti lengths and surface areas. While for low-vorticity flows in offshore domains the distortions to the triangles normally remain modest and the calculations are straightforward, several cases of realistic flows need particular attention (Figure 2). Firstly, if one or more particles forming a triangle gets stuck at the seashore (or drifts out of the model domain), the evolution of the triplet no more characterizes the compressibility of surface currents. To remove the associated non-physical values in Eq. (1), the further evolution of such triplets is discarded from when the trajectory of an element of the triplet lands on the shore.

Secondly (and usually more frequently) it may happen (especially in the Gulf of Finland that hosts strongly circularly polarised motions) that the three vertices of a triplet align along a straight line or coincide with each other. This would result in an attempt to divide by zero in Eq. (1). Physically, it means that very strong stretching occurs in the relevant area and the evolution of the triplet is no more able to properly characterise the flow compressibility. In order to avoid the resulting miscalculation, the evaluation according to Eq. (1) was stopped when the vertices became too closely aligned or nearly coincident. The further behaviour of such a triplet was ignored.

Although the final value of $C_{\text{pt}}$ for each point must be ≤1 (Giudici et al., 2012), there is still a possibility for single larger values of this measure at the first steps of calculation (Figure 3). Such values may appear in quite diverse locations (not shown) but they decrease rapidly, normally within one time step, into the expected range [0,1] (Figure 3). As a rule, the point-wise values of $C_{\text{pt}}$ for each point must be ≤1 (Giudici et al., 2012), there is still a possibility for single larger values of this measure at the first steps of calculation (Figure 3). Such values may appear in quite diverse locations (not shown) but they decrease rapidly, normally within one time step, into the expected range [0,1] (Figure 3). As a rule, the point-wise values of

![Figure 2. Potential failures of the calculation scheme in Eq. (1). (A): vertices of a triangular element tend to align, yielding a null area; (B): one of the vertices hits the coast.](image-url)
$C_{ftc}$ relax to the level close to their final values within 3–4 steps.

**RESULTS: SPATIAL DISTRIBUTIONS OF THE FINITE-TIME COMPRESSIBILITY**

The single estimates of the finite-time compressibility for different time spans were used to construct various maps characterising this measure for different sea areas and time intervals.

The overall average level of $C_{ftc}$ is around 0.4 during the windy season (October–March). Spatial distributions of the finite-time compressibility for single weeks (Giudici et al., 2012) and months (Figure 4) reveal comparatively large variability of this measure both in space and time. While for some months these distributions are almost featureless (like December 1987 in Figure 4), some other months contain a number of clearly defined areas with high values of $C_{ftc}$. The typical maximum values of $C_{ftc}$ for single months (like in February 1987 in Figure 4) are around 0.6, that is, well below the threshold $C_{ftc} = 0.7$ for the clustering process to become effective. The largest values of $C_{ftc}$ usually occur (i) in the nearshore of the southern coast of the Gulf of Finland, (ii) in the north-western part and (iii) in the easternmost domain of the gulf.

Offshore areas hosting largest monthly average values of $C_{ftc} > 0.5$ are normally different in different months (Figure 4). Only one area at the entrance to the River Neva estuary in the easternmost part of the gulf regularly hosts large values of finite-time compressibility. This domain hosts very small internal Rossby radius and contains an extremely complicated flow pattern driven by the interaction of basin-wide cyclonic circulation of the water masses and the optional anticyclonic gyre at the surface layer (Soomere et al., 2011) with the voluminous runoff of River Neva (Leppärinta and Myrberg, 2009). Another relatively persistent area of high values of $C_{ftc}$ is located in the middle of the widest part of the Gulf of Finland.

The seasonally averaged values of $C_{ftc}$ (Figure 5) are generally flatter and normally do not exceed the level of about 0.5.

Although it is likely that changes in the concentration occur at the levels of $C_{ftc}$ well below the threshold for clustering (see above), systematic patch formation can only be expected when the threshold of $C_{ftc} = 0.7$ is reached in some sea area for a relatively long time. The maps presented in Figures 4 and 5 give an overview of the spatial distribution of areas with high finite-time compressibility but they fail to deliver a good estimate of the likelihood of an area where clustering of particles or formation of high concentrations is likely to occur as the average $C_{ftc}$ tends to fade towards lower values when considering longer intervals (Giudici et al., 2012).

A more detailed insight into the temporal course of finite-time compressibility in selected domains (Figure 6) reveals that this measure normally fluctuates around its long-term value of ~0.4. Consistently with the information in Figures 4 and 5 it has more likely high values in the windy season than in the calm one. It exceeds the threshold for patch formation for many days in a row during several weeks (again more likely in the windy season). These weeks are, therefore, the most likely candidates for the patch formation process to become evident.

Given such an intermittent temporal behaviour of the finite-time compressibility, the temporal extent and spatial location of the areas prone to patch formation can be at best characterised using a method similar to the peak-over-threshold method used in many branches of physical oceanography, coastal engineering, and wave science (e.g., Holthuijsen, 2007). Following this idea, it was counted for each wet grid cell, how many instantaneous values of $C_{ftc}$ exceeding the critical level of 0.7 occurred during a certain time interval. The resulting maps (Figure 7) spot all the areas

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**Figure 3.** Dependence of finite-time compressibility (FTC) values initially >1 on the number of time steps in a calculation that started on the 1st of January 1987. The time step is 30 min.

**Figure 4.** Spatial distribution of finite-time compressibility (colour code) for the Gulf of Finland in December 1987 (upper panel) and February 1988 (lower panel).

**Figure 5.** Spatial distribution of finite-time compressibility for the Gulf of Finland in the windy season October 1987–March 1988. The average value is $C_{ftc} = 0.41$. Colour code is the same as for Figure 4.
which systematically show high \( C_{ftc} \), equivalently, the areas characterised by the frequent occurrence of favourable structures of currents for patch formation. The number of simulations (equivalently, days) in which the single values of \( C_{ftc} \) exceeds \( 0.7 \) are located outside the Neva Bight, for the first three months of the windy season (October 1987–March 1988, upper panel) and the calm season (April–September 1987, lower panel).

The majority of such areas are located in the nearshore. The largest area, characterised also by the largest amount of “days over threshold” stretches along the almost straight section of the southern coast of the Gulf of Finland. It is evident during both the windy and the calm season and very likely represents the presence of frequent downwelling owing to the predominant south-western winds (Lehmann and Myrberg, 2007). A similar area at the southern coast of Neva Bight in the easternmost domain of the gulf is located in very shallow waters and probably is not related to large-scale downwelling phenomena. Interestingly, it is present with more-or-less equal intensity in both calm and windy seasons. This feature suggests that it may be related with the interplay of the circulation in the gulf and the discharge from the River Neva. Several smaller nearshore areas located next to different peninsula may reflect zones of convergence of coastal currents with the basin-wide circulation.

Even more interesting is the presence of several offshore areas that frequently host high values of finite-time compressibility. Not unexpectedly, they are more evident during the windy season than in the calm period. Two areas are evident (with somewhat different percentage of cases with \( C_{ftc} \) over threshold) in both seasons. One of them is an elongated area at the entrance to Neva Bight in the eastern Gulf of Finland. As it was also highlighted in earlier calculations with a modest resolution (2 nm) of the ocean model (Giudici et al., 2012), it apparently reflects a very persistent feature of the dynamics of the Gulf of Finland (probably the presence of a more-or-less permanent convergence zone of the overall cyclonic circulation of the Gulf of Finland and the runoff of River Neva) and is the most probably candidate to provide frequent generation of patches of high concentration of different substances. The background for the similar area in the south-western part of the gulf and for the one that becomes evident only in the windy season in the middle of the widest section of the gulf needs to be clarified in the future.

### CONCLUDING REMARKS

The concept of finite-time compressibility of flow on sea surface combines the “ability” of the classical compressibility of 2D flows overlying 3D dynamics to serve as an indicator of the potential of formation of patches of high concentration of different substances with the history of Lagrangian transport of the affected counterparts. The definition of this measure (Eq. (1)) not only relates it to physical deformations of the sea surface but also allows to simply calculating its values in a systematic and consistent manner whenever the properties of surface currents are available with an appropriate resolution.

The presence of areas with persistently high values of finite-time compressibility in the southern nearshore of the Gulf of Finland, albeit interesting, is not completely unexpected because the areas where floating objects tend to gather are commonly associated with convergence zones and downwelling phenomena (Pichel et al., 2007; Samuelsen et al., 2012). This feature, however, points to the possibility to apply the technique of the evaluation of finite-time compressibility to operationally highlight the areas of frequent and strong downwelling directly from the map of surface velocities.

The presence of offshore areas frequently hosting high values of finite-time compressibility (except for possibly the one near the eastern end of the Gulf of Finland) seem to be a new dynamic feature of the circulation in this water body. It is highly interesting to independently verify their “ability” to systematically build large concentrations of tracers or substances.
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LITERATURE CITED


