Particles Deposition at Horizontal Flat Plate in Turbulent Particulate Flow

Alexander Kartushinsky,¹ Medhat Hussainov,¹ Efstathios E. Michaelides,² Ylo Rudi,¹ Igor Shcheglov,¹ Sergei Tisler¹* and Igor Krupenski¹

1. Research Laboratory of Multiphase Media Physics, Tallinn University of Technology, Akadeemia tee 15A 12618, Tallinn, Estonia
2. Department of Engineering, Texas Christian University, TCU Box 298640, Fort Worth, Texas 76129, U.S.A.

INTRODUCTION

Particulate flows are of high relevance in many engineering processes and natural phenomena, including sand storms, cosmic dusts, snow avalanches, the sedimentation of dust in the atmosphere. These flows are specially generated for industrial processes, which take place in power engineering, chemical, pharmaceutical, food and other branches of industry. The overwhelming majority of the above-mentioned natural phenomena and industrial processes are accompanied by the deposition of solid particles onto various surfaces that can be beneficial or harmful under the circumstances. Therefore, the understanding of the physical mechanisms that govern the deposition process is essential for the modelling of natural phenomena and optimal design of the industrial processes.

The problem of predicting the amount of solid particles depositing from the particulate flows onto various surfaces is of great interest for researchers. The investigation of the particles deposition have undoubtedly been stimulated by its practical relevance to many areas of technology and science, but the interest has also been aroused by the complexity of the problem and the inability of any theory to provide a truly satisfying physical explanation of the observed facts.

Numerous theoretical and experimental studies have been dedicated to the deposition of solid particles and the influence of the hydrodynamics of the particulate flows on the characteristics of these phenomena. However, most of them mainly deal with the theoretical aspects. The experimental studies do not consider the particles deposition as complex phenomenon, involving various physical processes that occur during the deposition.

Extensive experimental and computational studies related to the particles transport in the particulate flows have revealed the presence of the hydrodynamic boundary layer, which is formed close to the surface streamlined by the flow.¹–¹⁰ This boundary layer in many respects determines the particle deposition at the surface. Osiptsov,⁹ neglecting the inertial sedimentation, showed that the particles accumulated close to a body surface, and even for the insignificant values of the particles mass concentration the velocity field of a gas flow was distorted due to the accumulation of the particles near the surface. Soo¹¹ and Asmolov¹⁸ also observed the increase of the particles mass concentration in the vicinity of the surface of a flat plate in the laminar boundary layer. The experimental and theoretical results⁹,¹⁰ have revealed the maximum in the distribution of the particles mass concentration within the laminar boundary layer near vertical flat plate. These studies also showed that the amount of the deposited particles depends significantly on the value and location of this maximum. The mathematical model of the particulate laminar flat-plate boundary layer¹⁰ considered the inter-particle collision mechanism. Later, this model was developed by Kartushinsky and Michaelides.¹¹ Similar results were obtained by Wang and Levy¹²,¹³ for a turbulent flow over a flat plate and were attributed to the Saffman force and the particle–wall interactions.

Numerous studies show that the deposition process at the surface immersed in a particulate flow is determined substantially by the interrelationship between the properties of the carrier gas flow and the particles. The main parameters that characterise this relation

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Based on different assumptions, several models have been proposed to describe the deposition mechanisms in the turbulent flows. Friedlander and Johnstone\textsuperscript{15} were the pioneers introducing the model, where the particles were carried towards the vicinity of the wall by the turbulent diffusion and then reach the wall by free flight. The sublayer models were proposed by Cleaver and Yates\textsuperscript{16–18}, Fichman et al.\textsuperscript{19} and Fan and Ahmadi\textsuperscript{20} for the particle resuspension and deposition in the turbulent particulate flows. In the last work, the effect of a wall roughness was included, and an empirical equation for the particle deposition rate was proposed. Tsai and Lin\textsuperscript{21} proposed the numerical description of the particles deposition in a thermally driven natural convection for a vertical flat plate streamlined by the laminar flow, and reported about the attenuation of the deposition velocity with increase of the particles size.

The last two decades opened up the possibilities for more intensive studies of the near-wall particle–turbulence interactions by the direct numerical simulation (DNS) technique. These studies suggest that the particles deposition is mainly caused by the interactions between the particles and the coherent turbulent structures located in the near-wall region of the flow. This supports the hypothesis, first proposed by Cleaver and Yates,\textsuperscript{17} that the particles are convected to the wall if they are entrained by a fluid downwash toward the wall. This hypothesis is also found in the models developed by other authors, for example, Fan and Ahmadi\textsuperscript{22}.

The DNS modelling of the particles deposition in the wall-bounded turbulent aerosol flows were performed by McLaughlin\textsuperscript{23} and Ounis et al.\textsuperscript{24} Brooke et al.\textsuperscript{25} performed a detailed DNS study of the vortical structures in the viscous sublayer. Pedinotti et al.\textsuperscript{26} used the DNS technique to investigate the particle behaviour in the wall region of the turbulent flows. They reported that an initially uniform distribution of the particles tended to segregate into the low-speed streaks, and there is a resuspension of the particles being ejected from the wall. The DNS simulation was used by Soltani and Ahmadi\textsuperscript{27} to study the particle entrainment process in a turbulent channel flow. They found that the wall coherent structure played a dominant role on the particle entrainment process.

Squires and Eaton\textsuperscript{28} simulated the homogeneous isotropic non-decaying turbulent flow field by imposing an excitation at low wavenumbers, and studied the effects of inertia on the particles dispersion. They also used the DNS procedure to study the preferential micro-concentration structure of the particles as a function of the Stokes number in the turbulent near-wall flows.\textsuperscript{29} Rashidi et al.\textsuperscript{30} performed the experimental study of the particles turbulence interactions near a wall. They reported that the particles transport was mainly controlled by the turbulent burst phenomena.

Shams et al.\textsuperscript{31} used the Lagrangian simulation of transport and deposition of nano- and micro-particles (from 10 nm to 50 μm) including a Brownian motion effects in the sublayer of the turbulent boundary layer flow in the vertical and horizontal ducts. They extended the sublayer model for the turbulent deposition process to cover the effects of gravity, Brownian and lift forces. The model\textsuperscript{31} was based on the detailed analysis of the particle trajectories in the turbulence coherent structures near a wall. The Stokes drag, the Saffman lift and the Brownian excitation were included in the particle equation of motion. On the basis of the results\textsuperscript{31} one can draw two important conclusions: (1) the coherent near-wall eddies of the turbulent boundary layer flows play an important role in the particle deposition process; (2) the application of the sublayer model for the evaluating of the particle deposition is a reasonable approach.

Zhang and Ahmadi\textsuperscript{32} studied the particles deposition from the turbulent air streams in the vertical and horizontal ducts. They concluded that the particle-to-fluid density ratio, the shear-induced lift force, the flow direction and the shear velocity affect the particle deposition rate. For both vertical and horizontal ducts, the DNS results showed that the effect of gravity and its direction on the particle deposition rate became more significant at low shear velocities.

Zaichik and Alipchenkov\textsuperscript{33} developed a statistical model for the prediction of transport and deposition of high-inertia colliding massive droplets in the vertical pipe turbulent air flow. On the basis of analysis of the obtained results the authors drew the following conclusions: (1) the elastic collisions increase the deposition rate, while the inelastic collisions may result in a deposition decrease; (2) the decrease in the particle turbulence through the inelastic particle to particle collisions is not of great importance.

Marchioli and Soldati\textsuperscript{34} and Marchioli et al.\textsuperscript{35} using DNS for the study of the particles deposition in a fully developed turbulent open channel flow also revealed the strong accumulation of the particles very close to the wall in the form of the streamwise oriented streaks. This was attributed to the effect of turbophoresis, that is the preferential accumulation of particles at the regions of low turbulence.\textsuperscript{36}

Numerous recent numerical and experimental studies are pertinent to the particles deposition in the turbulent flows. Here should be mentioned the studies\textsuperscript{37–41} and many others, who considered various aspects of this rather complicated phenomenon.

The hydrodynamics of the particulate flow is only the first part of the deposition process. Along with purely hydrodynamic properties of a flow, there are other mechanisms that determine the particles deposition at the streamlined surface. After approaching a surface, the particles may be entrained and adhere to the surface or may rebound and be re-entrained by the gas flow. Therefore, the particle deposition process is an inherently probabilistic process that is governed by the relative strength of the adhesive and repelling forces. The detailed survey of these forces was carried out by Zimon,\textsuperscript{42} who was among the first to expound the fundamentals of the adhesion phenomenon. He explained that the adhesion process is very complex and depends on several factors, such as the particle–surface interaction, the properties of a particle and the surface (particle size, shape and surface roughness), the presence of electric charges and ambient conditions (humidity, temperature and chemistry). The probabilistic nature of the adhesion process, which takes place in the particulate flows, has been further examined by several recent studies\textsuperscript{43–47}.

Based on the short review of the state of the art of the particles deposition in various flows, one may draw the conclusion that there is a lack of experimental data, especially related to the deposition of particles on various obstacles streamlined by the particulate flows and embracing a wide range of the flow parameters and shapes. The unified models that would allow to predict the deposition for a wide range of the particulate flow regimes are still not available, except for some special efforts aimed...
at joining available models of the individual regimes. Efforts to improve such models are continuing, and there is a need for the experimental data that can be applied for the validation of such models.

In this paper, the solid particles deposition at the horizontal flat plate surface is studied for the moderate turbulent flow.

The choice of a flat plate was based on the simplicity of its surface shape, and that the hydrodynamic characteristics of the boundary layer, generated near a flat surface, had been already well studied for single-phase flow. The flat surfaces are the elements of the real equipment, for example ventilation ducts of the heating, ventilation and air conditioning (HVAC) systems, equipment of chemical industry, power engineering, etc., and the assessment of the deposition amount of solid particles and their distribution along flat surfaces is of primary importance in the optimal designing of such systems.

The numerical simulation of the particles deposition was accomplished for the laminar boundary layer developed near the flat plate surface. The two-fluid or two-existence approach was used for writing the Eulerian equations for the carrier gas and the particulate phase. The closure of the governing equations of the particulate phase based on the inter-particle collisions model by Kartushinsky and Michaelides\textsuperscript{[11]} and considering the motion of the real polydisperse mixture is presented in this paper. The solid particles were aluminium oxide powders of the multifractional composition. Along with the inter-particle collisions, other viscous drag and gravity forces, the lift Magnus and Saffman forces have been taken into account with considering the effect of the particle-fluid interaction on the carrier fluid motion. The lift Magnus force has been included into the model because of the particles interactions with the surface of the horizontal plate. Thus, along with the governing continuity, mass and momentum equations of gas and particles, the angular momentum equation written for the particulate phase was added to the combined equations. The system of the isobaric incompressible equations written in physical coordinates $x$ and $y$ for gas and particles was transformed to the proper form of the boundary equations with introducing of the self-similar transverse coordinate $\eta$.

The current model considers the particles deposition on a flat plate to be a probabilistic process, which is determined both by the hydrodynamics of the flow past the plate and the adhesive aspects of the particles deposition for its more accurate estimate that is suitable for various practical devices.

**MODEL DESCRIPTION**

The sketch of the computational flow domain within the laminar boundary layer generated near the flat plate is shown in Figure 1.

$$u(y)$$

Figure 1. Particulate flow within the flat-plate laminar boundary layer.

where $u(y)$ is the distribution of the longitudinal component of the gas velocity taken place across the boundary layer.

It is accepted that the particulate phase is polydisperse and composed of several mass fractions denoted by $\alpha_i$. It is assumed, without any loss in generality, that the mechanical effect of the size of each fraction can be characterised by the equivalent diameter of the $\alpha_i$ $D_i$.\textsuperscript{[48]} Three fractions are assumed to be presented in the formulation of the governing equations: for the 12 $\mu$m powder—9, 12, 15 $\mu$m (20%, 40%, 40% by mass, respectively) and for the 23 $\mu$m powder—18%, 23%, 28 $\mu$m (20, 40, 40% by mass, respectively).

System of Equations of Particulate Laminar Boundary Layer

The initial coordinate system of the boundary layer expressed in physical variables $(x, y)$ is transformed to the self-similar coordinate system $(\bar{x}, \eta)$, where $x$ and $\eta$ are calculated with applying of the transformation\textsuperscript{[49]} as:

$$\bar{x} = \frac{x}{L}, \quad \eta = \frac{y}{L} \sqrt{\frac{Re_x L}{x}}$$

The corresponding derivative transformations are as follows:

$$\frac{\partial}{\partial \eta} = \sqrt{\frac{Re_x}{x}} \frac{\partial}{\partial \bar{x}}, \quad \frac{\partial^2}{\partial \eta^2} = \left(\frac{Re_x}{x}\right) \frac{\partial^2}{\partial \bar{x}^2}$$

Then, the mass and momentum equations of gas and particulate phase written for the flat-plate laminar boundary layer and normalised to the length of the flat plate $L$ and the velocity of the free stream $U_\infty$\textsuperscript{[50]} are as follows:

The continuity equation:

$$\frac{\partial U}{\partial \bar{x}} + \frac{\eta}{2} \frac{\partial U}{\partial \eta} = 0 \tag{1}$$

The longitudinal momentum equation of gas:

$$\frac{\partial U}{\partial \bar{x}} + \left( V - \frac{U \eta}{2} \right) \frac{\partial U}{\partial \eta} = \frac{\partial^2 U}{\partial \eta^2}$$

$$= \sum_{i=1}^{3} \alpha_i \left[ x C^*_{Di} \frac{U - U_\infty}{\tau_{pi}} - C_{Mi} V - V_\infty \left( \frac{\nabla \times \bar{V}}{2} \right) \right]$$

$$- \sqrt{\frac{Re_x}{x}} \frac{\partial U}{\partial \bar{x}} \frac{\partial \omega_{h i}}{\partial \bar{x}} \tag{2}$$
The mass conservation equation of particulate phase:

\[
\frac{\partial \rho_{i} \mathbf{U}_{i}}{\partial t} + \frac{\partial}{\partial x} \left( \rho_{i} \mathbf{U}_{i} \mathbf{U}_{i} \right) = \frac{\partial}{\partial x} \left( \rho_{i} \mathbf{U}_{i} \mathbf{U}_{i} \right) - \mathbf{F}_{i} - \mathbf{G}_{i} - \mathbf{R}_{i}
\]

The longitudinal momentum equation of particulate phase:

\[
x_{U_{i}} \frac{\partial U_{i}}{\partial x} = \left[ \frac{\eta U_{i}}{2} + D_{i} \frac{\partial \ln \rho_{i}}{\partial x} \right] \frac{\partial U_{i}}{\partial x} - \frac{1}{\rho_{i}} \frac{\partial}{\partial x} \left( \rho_{i} \mathbf{V}_{i} \mathbf{V}_{i} \right) - \frac{\partial}{\partial x} \left( \rho_{i} \mathbf{V}_{i} \mathbf{V}_{i} \right)
\]

The transverse momentum equation of particulate phase:

\[
x_{U_{i}} \frac{\partial V_{i}}{\partial x} = \left[ \frac{\eta U_{i}}{2} + D_{i} \frac{\partial \ln \rho_{i}}{\partial x} \right] \frac{\partial V_{i}}{\partial x} - \frac{1}{\rho_{i}} \frac{\partial}{\partial x} \left( \rho_{i} \mathbf{V}_{i} \mathbf{V}_{i} \right) - \frac{\partial}{\partial x} \left( \rho_{i} \mathbf{V}_{i} \mathbf{V}_{i} \right)
\]

The angular momentum of particulate phase:

\[
x_{U_{i}} \frac{\partial \omega_{i}}{\partial x} + \left[ \frac{\eta U_{i}}{2} + D_{i} \frac{\partial \ln \rho_{i}}{\partial x} \right] \frac{\partial \mathbf{V}_{i}}{\partial x} = \frac{1}{\rho_{i}} \frac{\partial}{\partial x} \left( \rho_{i} \mathbf{V}_{i} \mathbf{V}_{i} \right) - \frac{1}{\rho_{i}} \frac{\partial}{\partial x} \left( \rho_{i} \mathbf{V}_{i} \mathbf{V}_{i} \right)
\]

Expression for the transverse velocity of gas obtained as the joint solution of the continuity and longitudinal momentum equations of gas:

\[
V = \sqrt{\rho_{i} U_{i}} = -U \int_{0}^{\Delta x} \left[ \frac{\partial}{\partial x} \left( \frac{\partial U}{\partial \eta} \right) - \sum_{i=1}^{3} C_{i} \frac{\partial \rho_{i} \mathbf{U}_{i}}{\partial \tau_{pi}} \right] d\eta / U^{2}
\]

The components of the velocities of gas and particles are presented in the dimensionless form as:

\[
\begin{align*}
\frac{U}{U_{i}} & = \frac{u}{U_{i}} \\
\frac{V}{U_{i}} & = \frac{v}{U_{i}} \\
\frac{\omega_{i}}{U_{i}} & = \frac{\omega_{i} L}{U_{i}}
\end{align*}
\]
Boundary Conditions

The boundary conditions written in the self-similar coordinate system are as follows:

\[ \eta = 0 : \ U = 0, \ V = 0, \ V'_{st} = (1 - k_n) V_{st}, \]
\[ \gamma' \sqrt{Re_{\infty}} \frac{\partial u_{st}}{\partial \eta} = -U_{st}, \ \lambda_i \sqrt{Re_{\infty}} x \frac{\partial \omega_{st}}{\partial \eta} = \bar{\omega}_{st}, \ \frac{\partial \alpha_i}{\partial \eta} = 0 \]

where \( k_n \) is the coefficient of restitution for the particle–wall collision, which, according to Matsumoto and Saito,[54] varies in the range of 0.92 ≤ \( k_n \) ≤ 1 and possesses the value of 0.92; the inter-particle distance \( \lambda_i \) is calculated as:

\[ \lambda_i = \frac{1}{2} \sqrt{\frac{\pi \rho}{\delta \rho_{st}}} - 1 \]

If the transverse velocity of the particulate phase \( V_{st} \) is negative, the correction[84] is used that considers the particle–wall collision. Thus, for the sliding collision, when \( |U_{st} - 0.5 \delta_{st}\bar{\omega}_{st}| > 3.5 \mu_0 (1 + k_n)|V_{st}| \), the boundary conditions are set as follows:

\[ U'_{st} = U_{st} + \mu_d \text{sgn}(U_{st} - 0.5 \delta_{st}\bar{\omega}_{st}) V_{st}, \]
\[ \bar{\omega}'_{st} = \bar{\omega}_{st} + 5 \mu_d \text{sgn}(U_{st} - 0.5 \delta_{st}\bar{\omega}_{st}) \frac{V_{st}}{\delta}, \quad V'_{st} = k_n V_{st} \]

where the static \( \mu_d \) and kinetic \( \mu_d \) friction coefficients possess the values of 0.6 and 0.4, respectively.

For the non-sliding collisions, when \( |U_{st} - 0.5 \delta_{st}\bar{\omega}_{st}| \leq 3.5 \mu_0 (1 + k_n)|V_{st}| \), the boundary conditions are set for the particulate phase at the wall as follows:

\[ U'_{st} = U_{st} - \frac{2}{7} (U_{st} - 0.5 \delta_{st}\bar{\omega}_{st}), \]
\[ \bar{\omega}'_{st} = \bar{\omega}_{st} + \frac{10 (U_{st} - 0.5 \delta_{st}\bar{\omega}_{st})}{7 \delta_{st}} \quad V'_{st} = k_n V_{st} \]

Here and above the primes denote that the corresponding variables are calculated for the post-collisional condition.

The present study does not consider the fluctuation velocity of the particles within the laminar boundary layer. Therefore, the gas and particles velocities and particles mass concentration are set equal to one at the border of the boundary layer, and these are the same values for all three particle fractions:

\[ \eta = \Delta \eta \sqrt{\frac{Re_{\infty}}{X}} : U = 1, \ U_{st} = 1, \ \alpha_i = 1, \]
\[ \frac{\partial V}{\partial \eta} = \frac{\partial V_{st}}{\partial \eta} = \frac{\partial \omega_{st}}{\partial \eta} = 0 \]
\[ \bar{x} = 0 : U = 1, \ V = 0, \ U_{st} = 1, \ V_{st} = 0, \ \bar{\omega}_{st} = 0, \ \alpha_i = 1 \]
\[ \bar{x} = 1 : \frac{\partial U}{\partial \bar{x}} = \frac{\partial V}{\partial \bar{x}} = \frac{\partial U_{st}}{\partial \bar{x}} = \frac{\partial V_{st}}{\partial \bar{x}} = \frac{\partial \omega_{st}}{\partial \bar{x}} = \frac{\partial \alpha_i}{\partial \bar{x}} = 0 \]

Numerical Method

The system of equations (Equations 1–6) was solved by means of the finite difference method by applying an upwind numerical scheme. The iterative procedure of the different rates of relaxation was used for the streamwise and transverse components of the momentum equations with applying the convergence criteria of 0.001 based on the least-square method.[55] The solution for the transverse velocity of the carrier gas was reduced to the quadratures (Equation 7) obtained by the joint solution of the continuity (Equation 1) and momentum (Equation 2) equations and integrated with the help of Simpson’s 3/8 rule[56] with 112 nodes over the boundary layer cross-section. The numerical solution was successfully compared with the classic profiles.[50]

Calculation of Particles Deposition

In order to compare the different experimental and numerical data on the particles deposition obtained for the different flow conditions, the results of the deposition experiments are usually presented as the slopes of the dimensionless deposition velocity \( V_{dep} \) versus the dimensionless particle relaxation time \( \tau_{p+} \). This is the time scale for the particle velocities to adjust to the surrounding flow velocity. By using the near-wall units, \( \tau_{p+} \) is defined according to Young and Leeming[57] as:

\[ \tau_{p+} = \frac{\tau_p u_v^2}{u} \]

where \( u_v \) is the friction velocity of gas.

Theoretical dimensionless deposition velocity is calculated according to Young and Leeming[57] as:

\[ V_{dep} = \frac{J_w}{u C_{\infty}} \]

where \( K_n \) is the probability of entrainment of particles by the surface, \( C_{\infty} \) is the density of the particulate phase in a free stream.

The deposition of solid particles in the laminar boundary layer at the surface of the vertical flat plate was studied in detail by Kartushinsky et al.,[10] where the hydrodynamic and adhesion aspects of the deposition phenomenon were investigated taking into account the probabilistic nature of deposition. Kartushinsky et al.[10] have derived the expression for the probability of entrainment of the particles by the surface \( K_{en} \), which considered the conditions of the particle–surface collisions, the adhesive properties of the given pair ‘particle–surface’ and the detachment of the deposited particles:

\[ K_{en} = \frac{1 - c_1 - c_2 \log \left( \frac{K_{el} \rho^2 V_{st \ imp}}{4} \right)}{1 - c_1 - c_2 \log \left( \frac{3 \pi \rho^2 V_{st \ imp}}{X} + \frac{K_{el} \rho^2 V_{st \ imp}}{4} \right)} \]

The first product term of the right side of Equation (10) describes the probability of adhesion of particles to the surface. The coefficients \( c_1 \) and \( c_2 \) are the empirical constants characterising the adhesive properties of the particles and surface; these coefficients are determined for the given pair ‘particle–surface’ by the experimental technique.[10] The constant \( k_{el} \) is determined by the elastic properties of materials of the surface and the particles. \( V_{st \ imp} \) is the particle impact velocity. The second product term of the right side of Equation (10) describes the probability of detachment of the deposited particle, which is determined by the blow-off due to the aerodynamic drag force and the elastic rebound. Here \( u_v \) is the gas flow velocity that occurs near the centre of the deposited particle. The dynamic friction coefficient \( \chi \), which
The coefficient $K_{en}$ is calculated in various locations $x$ along the flat plate surface for each particle size. The mass particles flux to the surface is defined as:

$$J_w = C_w v_{slw}$$

where the density of the particulate phase $C_w$ and the particle impact velocity $v_{slw}$ that take place immediately near the plate surface, are calculated by the present numerical model.

**EXPERIMENT**

The experimental setup for the investigation of the particles deposition in a horizontal turbulent flow is shown in Figure 2. The length of the test section was 2.5 m, the cross-section had the dimensions of 0.2 m $\times$ 0.4 m.

The smooth stainless flat plate of length $L = 0.5$ m, 0.1 m width and 0.002 m thickness was installed horizontally in the test section under zero incidence.

Turbulence of the gas flow was generated by the grid of 0.016 m mesh size and solidity of 0.34, and it was about 3.5% along the whole length of the flat plate for the free stream velocity $U_\infty = 5.1$ m/s.

Based on the LDA data of the longitudinal velocity obtained for the single-phase flow close to the plate surface (Figure 3), one can see that the generated boundary layer was the laminar. This is also confirmed by the calculation of the flat plate Reynolds number $Re_x$. According to Schlichting, if the flat plate at the zero incidence, the critical Reynolds number $Re_{x, crit}$ for the transition from the laminar boundary layer to the turbulent one is as follows:

$$Re_{x, crit} = \left( \frac{U_\infty x}{v} \right)_{crit} = (3.5 - 5) \times 10^5$$

This study was carried out for $Re_x$, which was about $2 \times 10^5$ at the location of the plate surface $x = 0.5$ m for $U_\infty = 5.1$ m/s.

According to Schlichting, Blasius’s law of friction can be applied to the single-phase flat-plate laminar boundary layer.

Then, the wall shear stress $\tau_{wx}$ occurring at the $x$ location of the flat plate surface is as follows:

$$\tau_{wx} = 0.332 v_U U_\infty \sqrt{\frac{U_\infty}{v x}}$$

Figure 3. The gas longitudinal velocity distributions within the boundary layer at various locations of the flat plate (1: $Re_x = 0.4 \times 10^5$; 2: $Re_x = 0.8 \times 10^5$; 3: $Re_x = 10^5$) and Blasius’ profile (4) occurring in the single-phase flow. Here $Y$ is the transverse coordinate of the boundary layer.

Ignoring any modification of the wall shear stress due to the presence of particles in the boundary layer, the friction velocity occurring at the $x$ location can be calculated as follows:

$$u_{*x} = \left( \frac{\tau_{wx}}{\rho} \right)^{1/2}$$

The aluminium oxide powders with the particle number average size $\delta_p = 12$ and $23 \mu$m were applied. The particle properties are presented in Table 1.

Table 2 presents the parameters of gas and particles calculated for the survey locations $x$ of the flat plate. Here $\Delta_{bx}$ and $\tau_p$, are the thickness of the boundary layer and the particles dimensionless relaxation time calculated at the location $x$, respectively. $\tau_{p,x}$ is calculated by Equation (8).

The particles were ejected into the flow by means of the L-shape narrow tube of diameter 0.005 m. The constant mass flux of the particles was obtained by means of the feeding device (Figure 2) and verified by the LDA system for every test run.

The particles longitudinal velocity in a free stream was measured in front of the flat plate upstream by the LDA separately from the main test runs. These measurements showed that both 12 and $23 \mu$m particles completely followed the gas flow.

Every test run included the simultaneous high-speed recording of the particles trajectories that was required for the calculation of the particles number concentration taken place in a free stream, the acquisition of images of the particles deposited at the survey surface area of the flat plate and the LDA control of the particles mass flux fed from the tubule.

The number concentration of the particles of a free stream was determined by means of the laser visualisation and high-speed video image-recording acquisition system (Figure 4) in the $(x, y)$ plane of the flow and by the subsequent analysis of the recorded images.
The sequence of the recorded frames was processed by the handler based on the LabView software. The result of this processing was the time sequence of the number of particles per each frame. Then, the particles number concentration $CN$ of a free stream was determined by the size of the measurement volume for every test run.

The measurements of the particles deposition were made at various locations $x$ of the plate. The amount of particles deposited at every location was measured by the laser illumination of the survey surface area with the help of the high-speed recording system (Figure 4). The duration of the image sequence was $t = 4.5\ s$ that assured that the particles covered the surface within the monolayer.

The obtained sequence of images was processed by the Scion Image software that allowed to count the number of particles $N_{\text{dep}}$ deposited at the survey surface area $S$ at the location $x$ of the plate surface. Then, the dimensionless particles deposition velocity $V_{\text{dep}+x}$ was calculated for the given location $x$ as:

$$V_{\text{dep}+x} = \frac{N_{\text{dep}}}{St} \frac{1}{CN U_{\infty x}}$$  \hspace{1cm} (15)

**RESULTS AND DISCUSSION**

The numerical results presented below have been obtained at various locations $x/L$ of the plate. The modelling considered separately the effects of gravity and the lift forces. The gas free-stream velocity $U_{\infty}$ was 5.1 m/s. The flow mass ratio of the aluminum oxide 12 and 23 μm particles was 0.07 kg dust/kg air, which made the flow a very dilute flow.

Figure 5 shows the comparison of the longitudinal gas velocity calculated within the flat plate boundary layer for the specific test case of the combined effect of the lift forces and gravity; Blasius’ gas velocity profile [59] is also presented. It is evident that for the given flow conditions the numerical distribution of the gas velocity coincides well with the classic profile by Blasius and demonstrates the laminar character of the flat-plate boundary layer, that was confirmed by the LDA data (Figure 3). The distributions of the particles longitudinal velocity $U_s$ calculated for the 12 and 23 μm particles are also presented in Figure 5. The velocity slip observed close to the wall (Figure 5) can be explained by the effect of inertia of the particles.

Figures 6–10 demonstrate the effect of gravity on the behaviour of the particles taken place within the boundary layer and the dimensionless deposition velocity with simultaneous neglecting of the lift forces.

<table>
<thead>
<tr>
<th>$\rho_p$ (kg/m$^3$)</th>
<th>3950</th>
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<tbody>
<tr>
<td>$\delta_p$ (μm)</td>
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</tr>
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<td>$\tau_p$ (s)</td>
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<td>$St$</td>
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<td>$C_{N_\infty}$</td>
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**Table 2. Deposition key parameters**

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<th>$x$ (m)</th>
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<th>0.18</th>
<th>0.30</th>
<th>0.39</th>
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</thead>
<tbody>
<tr>
<td>$Re_x$</td>
<td>$3 \times 10^5$</td>
<td>$9 \times 10^5$</td>
<td>$1.5 \times 10^5$</td>
<td>$1.9 \times 10^5$</td>
</tr>
<tr>
<td>$\Delta_{\text{kin}}$ (m/s)</td>
<td>0.022</td>
<td>0.0036</td>
<td>0.0047</td>
<td>0.0054</td>
</tr>
<tr>
<td>$\omega_x$ (m/s)</td>
<td>0.24</td>
<td>0.19</td>
<td>0.16</td>
<td>0.15</td>
</tr>
<tr>
<td>$\delta_p$ (μm)</td>
<td>12</td>
<td>23</td>
<td>12</td>
<td>23</td>
</tr>
<tr>
<td>$\tau_{\text{p}+x}$</td>
<td>6.8</td>
<td>24.9</td>
<td>4.1</td>
<td>15.0</td>
</tr>
</tbody>
</table>

![Figure 4](image-url). Particles concentration and deposition measuring systems.

**Figure 5.** Longitudinal velocity of gas $U$ (1: Blasius’ profile; 2: numerical calculation) and particles $U_s$ (3 and 4: 12 and 23 μm particles, respectively) within the boundary layer, location $x/L = 0.36$; the slip boundary condition is set for the particles; gravity and the lift forces are considered.

**Figure 6.** Effect of gravity: distributions of the transverse velocity of the 12 μm particles; the lift forces are neglected; 1: $x/L = 0.36$, gravity is considered; 2: $x/L = 0.36$, gravity is neglected; 3: $x/L = 0.68$, gravity is considered; 4: $x/L = 0.68$, gravity is neglected.
The profiles of the transverse velocity obtained at the locations $x/L = 0.36$ and 0.68 for the $12 \mu m$ and $23 \mu m$ particles, respectively, are shown in Figures 6 and 7, respectively. One can see that the effect of gravity on the particles motion is significant, and it is expressed in the change of direction of the transverse velocity to the opposite—from vectored to the outer boundary of the layer, when gravity is neglected, to vectored towards the plate surface, when gravity is considered. Moreover, the effect of gravity is the development of an apparent maximum disposed very close to the surface and increasing downstream. The rise of the particles size results in an increase of the absolute magnitude of the particles transverse velocity, which is obviously due to the particles inertia, and disappearance of its maximum (Figure 7).

Distributions of the particles mass concentration across the boundary layer (Figures 8 and 9) show the existence of the minimum, which disposes within the boundary layer, and the maximum, which locates either at the plate surface, when gravity is neglected, or immediately near the plate surface, when gravity is considered. Moreover, the effect of gravity is the development of an apparent maximum disposed very close to the surface and increasing downstream. The rise of the particles size results in an increase of the absolute magnitude of the particles transverse velocity, which is obviously due to the particles inertia, and disappearance of its maximum (Figure 7).

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Figure 10 shows the comparison of the numerical and experimental data on the dimensionless deposition velocity obtained with considering and neglecting of gravity with simultaneous neglecting of the lift forces. Here and below the dimensionless deposition velocity is plotted versus the dimensionless relaxation time of particles. It is evident that gravity allows to model satisfactorily the experimental data obtained for both particle sizes.

Figure 11 shows the combined effect of gravity and the lift forces on the particles mass concentration. One can see that all distributions have the pronounced maximum located near the surface. The magnitude of this maximum is proportional to the particles size. Also, the maximum falls inside the boundary layer downstream, and its magnitude grows.
As one can see from Figure 12, the combined effect of the lift forces and gravity gets the most reliable data for the particles deposition, which magnificently fit the experimental ones.

Figure 13 shows the distributions of the probability of entrainment $K_{en}$ calculated by Equation (10) for the 12 and 23 $\mu$m particles along the plate surface. Here the probability is presented versus the Reynolds number $Re_x$. It is evident that the probability of entrainment by the plate surface is higher for the 23 $\mu$m particles. This can be explained by the fact that the probability of adhesion (the first product term of the right side of Equation 10) is much more than the probability of detachment that arises from the predominance of the adhesive forces over the forces, which determine the elastic rebound from the surface and the blow-off due to the aerodynamic drag force.

Figure 14 presents the experimental and numerical data obtained for the combined effect of the lift forces and gravity. It is evident that the deposition velocity of the particles increases along the plate surface downstream. Besides, the deposition velocity of the 23 $\mu$m particles is larger as compared with the 12 $\mu$m particles. This fact can be explained by the appreciably higher concentration of the 23 $\mu$m particles, that takes place immediately near the plate surface (Figure 11). There is some discrepancy between the distributions of the deposition obtained by the experiments and by the presented numerical model. This is attributed to neglecting the adhesion effects, for example the influence of the electric charges of the particles and surface, as well as the ambient conditions (humidity, temperature and chemistry).

CONCLUSIONS

The deposition of aluminium oxide 12 and 23 $\mu$m particles onto the horizontal flat plate occurring in the laminar boundary layer in the free-stream moderate grid-generated turbulent flow was studied, both numerically and experimentally. The presented numerical model considers the particles deposition on a flat plate to be a probabilistic process, which is determined both by the hydrodynamics of the flow past the plate and the adhesive behaviour of particles and the plate surface. The numerical simulation of the particles deposition was realised in the laminar boundary layer that was generated near the plate surface. The two-fluid approach was used for writing the Eulerian equations for the carrier gas and particulate phase.

The effects of gravity and the lift forces on the particles longitudinal and transverse velocities, mass concentration and the particles deposition, which take place within the flat-plate boundary layer, were studied separately for various particle sizes.
It was observed that gravity and the lift forces have decisive influence on the behaviour of solid particles within the boundary layer and, therefore, on their deposition, that is expressed via the distributions of the particles transverse velocity and mass concentration. These effects become more pronounced for the larger 23 µm particles. This fact, coupled with the higher probability of entrainment by the surface, results in their larger deposition velocity.

The numerical results show that the effect of gravity is stronger than the effect of the lift forces, that expresses in the particles behaviour within the boundary layer and their deposition at the plate surface.

Thus, the obtained numerical results show that the deposition process is determined both by the hydrodynamics of streamlining of the plate surface (the distributions of the particles mass concentration and the transverse velocity of particles taken place close to surface) and the elastic and adhesive properties of the particles and the surface.

NOMENCLATURE

- $C_N$: number concentration of particles (1/m$^3$)
- $C_D$: relative viscous drag coefficient of a particle
- $C_F$: coefficient of the Saffman lift force
- $C_M$: coefficient of the Magnus lift force
- $C_w$: torque coefficient
- $C_{\infty}$: density of particulate phase in a free stream (kg/m$^3$)
- $c_1$: constant (see Equation 10)
- $c_2$: constant (see Equation 10)
- $D_s$: pseudodiffusivity coefficient
- $E^g$: elliptic integral of the second order
- $F_{ID}$: aerodynamic drag force (N)
- $F_G$: gravity (N)
- $F_{LR}$: lift Magnus force (N)
- $F_{LS}$: lift Saffman force (N)
- $f$: friction coefficient for particle–particle collision
- $g$: gravitational acceleration
- $J_{wp}$: mass flux of particles to the plate surface (kg/(ms$^2$))
- $K$: integral of the first order
- $K_{en}$: probability of entrainment of particles by the surface
- $K_{el}$: elliptic integral modulus
- $k_{el}$: constant determined by the elastic properties of materials of surface and particles
- $k_{pt}$: coefficient of restitution for particle–wall collision
- $k_{pm}$: coefficient of restitution for particle–particle collision
- $L$: length of flat plate (m)
- $N_{dep}$: number of the deposited particles
- $Re_s$: particle Reynolds number
- $Re_\infty$: Reynolds number of gas free stream
- $S$: survey surface area (m$^2$)
- $St$: particle Stokes number
- $St_{cr}$: critical particle Stokes number
- $t$: time period of observation(s)
- $t_{col}$: inter-particle collision time
- $U$: dimensionless longitudinal component of gas velocity
- $U_s$: dimensionless longitudinal component of velocity of particulate phase
- $u_\infty$: free-stream velocity (m/s)
- $u$: longitudinal component of gas velocity (m/s)
- $u_s$: gas flow velocity near the centre of the deposited particle (m/s)
- $u_t$: longitudinal component of velocity of particulate phase (m/s)
- $V_{dep+}$: dimensionless deposition velocity
- $V$: dimensionless transverse component of gas velocity
- $V_r$: dimensionless slip velocity
- $V_s$: dimensionless transverse component of velocity of particulate phase
- $v$: transverse component of gas velocity (m/s)
- $v_s$: transverse component of velocity of particulate phase (m/s)
- $v_{imp}$: particle impact velocity (m/s)
- $x$: longitudinal coordinate (m)
- $\chi$: reduced longitudinal coordinate
- $y$: transverse coordinate (m)

Greek Symbols

- $\alpha$: mass concentration of particulate phase (kg dust/kg air)
- $\chi$: dynamic friction coefficient
- $\Delta$: thickness of laminar boundary layer (m)
- $\delta$: equivalent diameter of particle (µm)
- $\delta_{\infty}$: dimensionless particle size
- $\delta_p$: particle number average size (µm)
- $\psi$: collision angle (rad)
- $\lambda$: inter-particle distance
- $\eta$: self-similar transverse coordinate
- $\mu_d$: kinetic friction coefficient
- $\mu_s$: static friction coefficient
- $v$: kinematic viscosity of gas (m$^2$/s)
- $\nu_s$: pseudoviscosity coefficient
- $\rho$: physical density of gas (kg/m$^3$)
- $\rho_p$: material density of particles (kg/m$^3$)
- $\tau_p$: particle relaxation time (s)
- $\overline{\tau}_p$: normalised particle relaxation time
- $\tau_{p+}$: dimensionless particle relaxation time
- $\tau_w$: wall shear stress (N/m$^2$)
- $\omega_s$: angular component of velocity of particulate phase (rad/s)
- $\psi_i$: correction coefficient of the Saffman lift force

Subscripts

- $i$: refers to the particles fraction
- $j$: refers to the particles fraction
- $k$: refers to $x$- or $y$-projection of lift force
- $p$: refers to the properties of a single particle
- $s$: refers to the properties of particulate phase
- $x$: refers to the streamwise direction
- $w$: refers to the properties that take place near the plate surface

Superscripts

- $'$: refers to post-collisional conditions

ACKNOWLEDGEMENTS

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The magnitude of the translational velocity of the colliding particles is much smaller than the angular velocity:

\[
\langle A^2 \rangle_{\psi,x,\theta} = \frac{1}{3} + \frac{f^2}{10} , \quad \langle B^2 \rangle_{\psi,x,\theta} = \frac{1}{12} + \frac{2f^2}{5}.
\]

\[
\langle AB \rangle_{\psi,x,\theta} = \langle AC \rangle_{\psi,x,\theta} = 0 , \quad \langle BC \rangle_{\psi,x,\theta} = -\frac{2f^2}{5}.
\]

\[
\langle C^2 \rangle_{\psi,x,\theta} = -\frac{5f^2}{12}.
\]

(2) The translation velocity of the colliding particles is of the same order of magnitude as the angular velocity:

\[
\langle A^2 \rangle_{\psi,x,\theta} = 1 - \frac{t}{8} + \frac{f^2}{6} , \quad \langle B^2 \rangle_{\psi,x,\theta} = \frac{1}{12} + \frac{t}{16} + \frac{f^2}{6}.
\]

\[
\langle AB \rangle_{\psi,x,\theta} = \langle AC \rangle_{\psi,x,\theta} = 0 \quad \text{and} \quad \langle BC \rangle_{\psi,x,\theta} = -\frac{f}{5} + \frac{t}{3}, \quad \langle C^2 \rangle_{\psi,x,\theta} = \frac{f^2}{4}.
\]

where \( f \) is the friction coefficient.

The other coefficients of Equations (A1)–(A4) are as follows:

\[
A_{ij} = \frac{\varphi_{ij}}{E^2} \left[ 1 - \frac{k_{ij}^2}{2} \left( 1 + \sin \varphi_{ij} \right) \right],
\]

\[
B_{ij} = \frac{k_{ij}^2 \sin^2 \left( \varphi_{ij}/2 \right)}{E^4} \left[ 1 - \frac{V_j}{V_i} \cos \varphi_{ij} \frac{\varphi_{ij}}{2} \right], \quad C_{ij} = \frac{k_{ij}^2 V_i \varphi_{ij}}{4V_i E^2} \left( 1 - \frac{\sin 2\varphi_{ij}}{2\varphi_{ij}} \right),
\]

\[
L_{ij} = k_{ij}^3 \frac{\sqrt{V_j}}{V_i} \left[ \sin \varphi_{ij} \frac{\sin \varphi_{ij}}{E^2} \right] \left[ 1 - \frac{1}{3k_{ij}} \frac{V_j}{V_i} \right],
\]

\[
M_{ij} = \frac{1}{3E^2} \left[ 1 - \frac{1}{1 - k_{ij}^2 \cos^2 \left( \varphi_{ij}/2 \right)} \right]^{1/2}.
\]

Here \( \gamma_i \) is the maximum value of the collision angle.

The coefficients of the inter-particle collisions model \( A, B, \) and \( C \) were calculated by the averaging of products of differences of the linear and angular velocity components of the colliding particles (Equations A1–A3), which was carried out over the coordinates of collision \( (\psi, x, \theta) \) for the next cases:

(1) The magnitude of the translational velocity of the colliding particles is much smaller than the angular velocity:

\[
\langle A^2 \rangle_{\psi,x,\theta} = 1 - \frac{2f}{10}, \quad \langle B^2 \rangle_{\psi,x,\theta} = \frac{1}{12} + \frac{2f^2}{5}.
\]

\[
\langle AB \rangle_{\psi,x,\theta} = \langle AC \rangle_{\psi,x,\theta} = 0, \quad \langle BC \rangle_{\psi,x,\theta} = -\frac{2f^2}{5}.
\]

\[
\langle C^2 \rangle_{\psi,x,\theta} = -\frac{5f^2}{12}.
\]
The collision angle $\varphi_{ij}$ is defined as:

$$\varphi_{ij} = \left| \arctg \left( \frac{(v_{ix}/u_{ix}) - (v_{iy}/u_{iy})}{1 + (v_{ix}/u_{ix}) (v_{iy}/u_{iy})} \right) \right|$$

The ratio between the transverse and longitudinal components of the particles velocities is as follows:

$$\tan \gamma_i = \frac{v_{ix}}{u_{ix}}$$

The coefficient $\beta_{ij}$ is the ratio of masses of the colliding particles, $\beta_{ij} = (m_i/(m_i + m_j))$. The coefficient $a$ is calculated via the restitution coefficient of the colliding particles as: $a = 1 + k_{em}$.

REFERENCES


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