Sound reflection at an open end of a circular duct exhausting hot gas

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Sound reflection from hot flow duct openings is a classical problem in acoustics. In practice this is important for effective modelling and prediction of noise radiation from engine exhaust systems, burner pipelines, exhaust nozzles etc. Despite several experimental and theoretical investigations in the field, there is still limited experimental data available to validate the existing theory. In the present study, experimental investigations of plane acoustic wave reflections at duct openings where a hot jet flows into relatively cold surrounding media have been carried out. Heated air with well determined and homogenous chemical consistency along the duct axis was used as a testing media inside the duct during the experiments. The studied jet temperatures exhausting from the pipe ranged from room temperature up to 500 °C. The standard two-microphone technique was applied to determine the reflection properties at the duct opening. The experimental results for the reflection coefficient magnitude and phase have been compared with Munt’s theory and good correlation was found. This result is a first experimental validation of the theory for hot flow conditions.

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1 INTRODUCTION

Despite the fact that the noise radiating from a duct exhausting hot gas to the atmosphere is a common problem associated with IC-engines exhaust systems, burner stacks and aeronautical jet engine applications, it has received little attention.

A classical theoretical work on the sound radiation from an open end of straight pipe has been performed by Lewine and Schwinger\textsuperscript{1} for no flow condition. Another theoretical treatment of the radiation of sound from duct opening, limited to semi-infinite, thick-walled circular duct radiating into an anechoic space, was conducted by Ando\textsuperscript{2}. According to this theory the radiation impedance depends only on the Strouhal number \( k \cdot a / M \) for a given wall thickness. Besides the fact that in all the real experiments the length of the duct is finite and often opens into spaces which, at least over a part of frequency range, are not perfectly anechoic, the theory is further limited in cases where significant temperature difference between the gas flowing from the duct and that of the surrounding air exists, as considered in the present paper.

A major theoretical work on the radiation of sound from an open ended straight pipe with flow has been performed by Munt\textsuperscript{3,4}. The mathematical model involves the use of the Wiener-Hopf technique and a full Kutta condition. The theory of Munt\textsuperscript{3,4} predicts that the reflection coefficient magnitude can exceed unity within a critical range of Helmholtz number \( k \cdot a \). Cargill\textsuperscript{5} derived analytical expressions for the far field radiation and pressure reflection coefficient at the opening of a flow pipe. For the case of low Strouhal number a similar behavior for the reflection coefficient magnitude was found when the Kutta condition was imposed at the edges of pipe end.

There have been attempts\textsuperscript{6–8} to experimentally investigate the radiation properties of ducts exhausting hot gas using a more time consuming and technically demanding standing wave ratio technique\textsuperscript{9}. A wide temperature range (18 °C–1000 °C) was investigated by Fricker and Roberts\textsuperscript{7} by conducting the experiments with a flow of hot combustion gases. The flow velocity in the duct was low \((M < 0.03)\) during these experiments. The measured results showed considerable scatter which was believed\textsuperscript{8} to originate from the thermal effects affecting the functioning of the acoustic pressure probe. This problem can occur and cause a decrease in accuracy when the sensitivity of the probe is not constant at positions with different temperatures in ducts with hot gas, when implementing the standing wave ratio technique\textsuperscript{9}.

An issue to be considered when burners are used to

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create the hot flow is related to the properties of the jet. In such the experimental set-up the jet is formed by combustion products exhausting from the open end of the pipe. This makes the chemical consistency of the jet potentially less determined and less uniform as compared to electrically heated air for which the properties required for modelling can be accurately controlled for any temperature condition. In previous works (e.g. by Cummings6) the alternative to using heated air has successfully been implemented for similar purposes although in a rather limited temperature range (up to 300 °C). When implementing a theoretical model (e.g Munt’s model4) for a comparison with the experimental results, knowledge about the exact properties of the flowing gas leaving the pipe and of the media surrounding the pipe is crucial as these properties are used to determine the primary input data for the model (such as the density and the speed of sound of the fluids).

Additional problems to overcome when testing with high temperature jet, as reported in earlier experimental works6–8, have been related to temperature gradients in a test section of the duct which typically lead to a distorted pressure wave pattern and a loss of accuracy in results unless properly compensated for. When testing in heated air-flow conditions, Cummings6 incorporated electrical heating elements together with a thermal isolation layer outside the test section to minimize axial and radial temperature gradients. He also took care of the pressure probe sensitivity problem by using a double-walled glass tube kept at the same temperature during the experiments. His results show much less scatter and it was demonstrated that high temperatures have a marked effect on sound radiation from a duct.

The aero-acoustic behaviour of an open pipe has been investigated experimentally with a multi-microphone technique by Peters et al10.

Some recent studies on sound reflections at duct openings have mainly been focused on the systematic investigation of different opening geometries11 or the determination of acoustic pressure reflection coefficient in case of mean flow12 using plane wave decomposition in ducts. Because of its frequent citation and its status of forming the bases for experimental and theoretical results in most of the contemporary publications on open-ended duct terminations (e.g. by Peters et al10 and Allam and Åbom12) there is a reason to believe that Munt’s model4 is presently the most refined and accurate.

Munt’s model describes the propagation of acoustic waves out of the semi-infinite circular pipe in the presence of subsonic flow. The flow jet is assumed to leave the pipe uniformly with a thin cylindrical boundary layer. The pipe is assumed to have rigid walls of negligible thickness, described by polar coordinates \((r, \theta, z)\), see Fig. 1. The fluid inside the jet is taken to have density \(\rho_j\), the speed of sound \(c_i\) and the fluid velocity \(\mathbf{M}\cdot c_i\). The ambient gas is taken to be stagnant or flowing in the same direction with the jet. It is also taken to have a density \(\rho_s\), speed of sound \(c_i\) and fluid speed \(\mathbf{M}\cdot c_s\), where \(\mathbf{M} = C\) are non-dimensional quantities that determine the ratios of the ambient gas values to the jet values. The gas jet leaving the pipe termination is assumed to have a Mach number \(M < 1\).

In the Munt’s model4 a full Kutta condition at the pipe opening is applied. This implies smooth separation of the flow and no pressure fluctuations at the edge. The model involves the infinitely thin vortex sheet separating the flow jet from the ambient gas at \(r > a\). Instability in the sheet is excited by the sound transmitted through the open end. This instability radiates sound and creates an additional sound field in the jet pipe. The additional sound field is taken into account and is a consistent part of the derived model4. The solutions for the acoustic field in the pipe and in the exterior jet flow are not subject to any approximations unlike in the earlier works5 where certain approximations were used to solve the mathematical model.

The sound in the pipe propagating towards the open end is being altered and reflected back by both the pipe termination and by the instable vortex sheet formed outside the pipe open end. The acoustic reflection coefficient of the pipe termination is determined by the reflected acoustic wave relative to the incident wave.

It is important to mention that the Munt’s model4 can predict values for the plane wave reflection coefficient...
that exceed unity at low frequencies and $M \neq 0$, in accordance with the experimental observations in earlier investigations.

Munt’s model has been validated only for the mean flow conditions up to $M < 0.2$ and $k \cdot a < 1.3$ in case of air flow at room temperature by the published experimental data. The purpose of our study was to perform precise measurements of the acoustic pressure reflection from an open ended pipe exhausting high temperature (up to 500 °C) gas and to validate Munt’s model.

The solution for Munt’s model is expressed in the form of integral equations which are solved numerically. In order to obtain the theoretical pressure reflection coefficients presented in this paper for comparison with the experimentally determined results a MATLAB model coded by Panhuis has been implemented. This model calculates the complex valued reflection coefficient $R$ at duct opening presented in the two papers written by Munt.

2 METHOD

The procedure used in the current paper to experimentally determine the reflection coefficient of the duct opening follows the two-microphone method, which is shortly described below. The plane wave acoustic pressures at two microphones (pressure transducers for the actual case) separated by distance $s$ (see Fig. 2) in the duct wall can be expressed as

$$p_1(f) = p_{1+}(f) + p_{1-}(f),$$

$$p_2(f) = p_{1+}(f)e^{-ik_1s} + p_{1-}(f)e^{ik_1s},$$

where $p$ is the acoustic pressure, $f$ is the frequency, $k = 2\pi f/c$ is the wave number, $c$ is the speed of sound, $+$ and $-$ denote the pressure waves propagating in neg. and pos. direction relative to the x-axis, $k_1 = k/(1-M)$, $M$ is the Mach number. The reflection coefficient at transducer 1 is defined as

$$R_1(f) = p_{1+}(f)/p_{1-}(f),$$

where $r$ denotes the values measured at transducer 1 cross-section and $p_{1+}$, $p_{1-}$ is the acoustic field generated by the external source (electro-dynamic driver). From equations (1)–(3) it can be shown that

$$R_1(f) = (H_{12} - e^{-ik_1s})/(e^{ik_1s} - H_{12}),$$

where $H_{12} = h_2/h_1$ is the transfer function from pressure transducer 1 to 2 and $h_1 = p_1/e$ and $h_2 = p_2/e$ are the transfer functions between the pressure transducer signals $p_1$, $p_2$, and the reference signal (here the electrical signal $e$ driving the electro-dynamic driver), respectively.

The reflection coefficient $R_1$ is then transferred to the duct termination cross-section by

$$R = R_1 e^{-ik_1l},$$

where $l$ is the distance from the transducer 1 cross-section to the duct termination.

3 EXPERIMENTS

Experiments were performed using a steel automotive exhaust pipe. The pipe had a circular cross-section, inner diameter $d = 41.0$ mm, and total length $l = 1.2$ m. The wall thickness was $\Delta = 2.0$ mm. The experimental set-up used for the measurements is presented in Fig. 3. During the experiments the test duct was positioned vertically, opening directed upwards in order to achieve a symmetrical and uniform outflow condition to a surrounding environment (air at room temperature, 21 °C).

A pressure transducer separation $s = 0.07$ m was chosen for experiments where the jet temperatures ranged from 21 to 500 °C. Boden and Åbom proved that the two microphone method has its lowest sensitivity to errors in the input data in a region around $k_0 < \pi(1-M^2)/2$. Also Åbom and Bodén suggested that to avoid a large sensitivity to errors in the input data, the two microphone method should be restricted to the frequency range: $0.1 \pi(1-M^2) < k_0 < 0.8 \pi(1-M^2)$. Accordingly, the frequency region reckoned during the measurements varied from 245 Hz...1.9 kHz (for room temperature air at 21 °C) to 391 Hz...3.1 kHz.
(heated air at 500 °C). The first cut-on frequency of this circular duct \( f_C = 1.841 \cdot \frac{c}{d} \cdot \frac{1}{2} \) varied from 4.9 kHz \( (t=21 °C) \) to 7.8 kHz \( (t=500 °C) \) respectively. To reduce the random error in the measurements a relatively large number of averages (300 for all the temperatures) was used.

Two piezoelectric pressure transducers (Kistler 4045A) equipped with water-cooling mounts were used in the test pipe to simultaneously record the acoustic pressure signals (see Fig. 3). The transducers were located on the opposite duct walls in order to suppress uncorrelated flow disturbances at the walls. The pressure signals were conditioned and amplified by piezo-resistive amplifiers (Kistler 4603). A random acoustic excitation was provided by electro-dynamic driver (DAS ND-8) mounted to a side-branch of the test duct (see Fig. 3). The electro-dynamic driver was driven by a software-based signal generator (NCH) through a power amplifier (Velleman VPA2100MN). The signal acquisition was performed by a four-channel dynamic signal analyzer (National Instruments NI PCI-4474). The analyzer was controlled by a custom built PC based virtual instrument (LabView).

A relatively low flow velocity \( v=2.5 \text{ m/s} \) (Mach number \( M < 0.01 \)) was used for all the jet temperatures studied during the experiments in order to minimize the flow velocity effects on the results. The flow velocity was determined by a portable anemometer (Delta Ohm HD2134P.2) using a Pitot tube temporarily mounted to the duct opening.

The hot air flow was provided by two electrical blowers (Makita HG650C) equipped with automatic temperature and flow velocity control module. The blowers were mounted in parallel to the inlet side of the test duct. In order to minimize the axial and radial temperature gradients in the duct as well as to achieve higher jet temperatures, seven surface heaters: four 550 W heaters (Bencon 6009) and three 250 W heaters (Bencon 5984) were mounted to the external surface of the duct. The heaters were driven by automatic temperature controller (Omron E5CN) via a solid state relay (Omron G3PB).

Six custom made K-type temperature sensors (0627 1134-03 and 0627 1134-04) were mounted to three different cross-sections of the duct. Temperature was measured in both pressure transducer cross-sections and in the cross-section 10 mm from the duct termination. In each cross-section the radial positions of the temperature sensors were determined considering the equivalent flow areas in that cross-section. After the temperature had stabilized in the test section the temperature values were obtained by dual K-type thermometers (TES-1312). These temperature values were recorded and averaged in order to determine the

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**Fig. 3—Experimental set-up for the measurements. Sketch (left side) and photo (right side) of the test rig.**
speed of sound in the jet. The local temperature readings were furthermore taken into account to regulate the heaters with the aim to minimize axial and radial temperature gradients.

4 RESULTS AND DISCUSSION

The experimentally determined and theoretically predicted magnitude and phase of the plane wave reflection coefficient are presented in Figs. 4–15. Since the flow velocity did not exceed 2.5 m/s during the experiments the results presented here can be regarded as $M=0$ case.

The reflection coefficient magnitudes for several jet temperatures (from 21 °C to 500 °C) are presented in Figs. 4, 6, 8, 10, and 12. It can be seen that for $ka=0$ the theoretical value for the magnitude approaches unity. The same trend was also predicted by the Munt’s model\(^4\) and confirmed by several other authors\(^{10–12}\) for any flow Mach number, both experimentally and theoretically. The magnitude decreases with increasing Helmholtz number $ka$ and does not exceed unity in any temperature condition tested. Values above unity have previously been observed\(^4,12\) for room temperature flow conditions in cases with considerable flow velocity difference between the exhausting jet and the ambient gas.

This result shows that without a remarkable mean flow the temperature gradient across the shear layer itself does not cause the transfer of acoustic energy into the kinetic energy in the shear layer. This kind of energy transfer has been reported\(^4\) to explain the hump of the reflection coefficient magnitude, exceeding unity.

Fig. 4—The magnitude of the reflection coefficient for open-ended duct; measured at $t=100$ °C, (green stars), predicted (black dashed line).

Fig. 6—The magnitude of the reflection coefficient for open-ended duct; measured at $t=200$ °C, (green diamonds), predicted (black dashed line).

Fig. 5—The phase of the reflection coefficient for open-ended duct; measured at $t=100$ °C, (green stars), predicted (black dashed line).

Fig. 7—The phase of the reflection coefficient for open-ended duct; measured at $t=200$ °C, (green diamonds), predicted (black dashed line).
at certain Helmholtz numbers (ka), in case of flow velocity difference adequate to cause vortex shedding in the shear layer.

The figures show that the experimental results have a good correlation with the theoretical ones and the discrepancy does not exceed 5–6% over the ka range investigated.

The reflection coefficient phase for several jet temperatures are presented in Figs. 5, 7, 9, 11, and 13. For ka=0 the theoretical value of the phase attains the value of \( \pi \) and decreases with increasing ka. Such a behaviour has also been observed by several theoretical and experimental investigations\(^4,12\) in room temperature conditions. The experimentally determined phase shows a good correlation with the theoretical values for all the temperature conditions tested. The discrepancy is in general below 4–5%, except in case of 500 °C where it reaches around 10% at higher ka values. The larger discrepancies at higher temperatures can probably be regarded as measurement uncertainties due to the appearance of the axial and radial temperature gradients along the pipe. Despite of attempts to minimize the temperature gradients (by using surface heaters and isolation layers in critical areas) they were evident at higher jet temperatures and could lead to a less accurate values for speed of sound.

It is worth noting here that the theory presupposes a cylindrical jet from the opening separated from the surrounding media by a thin shear layer. Though this may not be completely fulfilled for cases of low Mach number flows as studied in these experiments, the prediction accuracy was nevertheless good.

For a comparison, the reflection coefficient for
different jet temperatures are plotted as a function of the Helmholtz number \( ka \) in Figs. 14 and 15, including the results measured in room temperature conditions. From the Fig. 14 it is evident that, for any value of \( ka \), the magnitude of the reflection coefficient decreases as the temperature is increased. In practice this result means that more acoustical energy (thus more noise) is being transmitted into the surrounding fluid by the jet at higher temperatures. As one can see from the Fig. 15, the theoretically obtained phase of the reflection coefficient drops slightly more steeply from its initial value of \( \pi \) in case of decreased temperature, whereas this phenomena is not as distinct in the experimental curves due partly to the aforementioned measurement errors.

The observed decrease in the reflection coefficient with increased temperature can be explained by the fluid density gradient across the shear layer of the hot exhausting jet and the relatively cold surrounding media. Here the hot jet virtually forms a continuation of the duct at the open end with less of a sudden change in wall impedance than would occur in homogeneous fluid around the duct termination. This causes more transmission and less reflection of the acoustic energy back into the pipe.

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**Fig. 12**—The magnitude of the reflection coefficient for open-ended duct; measured at \( t=500 \, ^\circ \text{C} \), (green triangles), predicted (black dashed line).

**Fig. 13**—The phase of the reflection coefficient for open-ended duct; measured at \( t=500 \, ^\circ \text{C} \), (green triangles), predicted (black dashed line).

**Fig. 14**—The magnitude of the reflection coefficient for open-ended duct; measured at \( t=21 \, ^\circ \text{C} \), (blue triangles), predicted (blue dash-dotted line), measured at \( t=200 \, ^\circ \text{C} \), (green diamonds), predicted (green dashed line), measured at \( t=400 \, ^\circ \text{C} \), (red squares), predicted (red solid line).

**Fig. 15**—The phase of the reflection coefficient for open-ended duct; measured at \( t=21 \, ^\circ \text{C} \), (blue triangles), predicted (blue dash-dotted line), measured at \( t=200 \, ^\circ \text{C} \), (green diamonds), predicted (green dashed line), measured at \( t=400 \, ^\circ \text{C} \), (red squares), predicted (red solid line).
5 CONCLUSIONS

A number of experiments have been performed with jets with various temperatures up to 500 °C, exhausting from a circular tube opening into relatively cold media.

The results obtained agree well with the theoretical model developed by Munt and hence it has been demonstrated that the theory can be used with reasonable accuracy to predict the sound reflection from the open pipe termination in high jet temperature conditions and low Mach and Helmholtz numbers. It was found that the magnitude of reflection coefficient decreases significantly with increased temperature and does not exceed unity at Helmholtz numbers up to 0.8 in cases of negligible mean flow. The quantitative data presented in this paper for the reflection coefficients should be directly applicable for high temperature gas jet outlets within a reasonable accuracy.

An interesting continuation for the research would be to perform the high temperature experiments with considerably higher flow velocities, in order to include the Mach number effects. Also it is of interest to test different opening geometries that are often used for practical applications (bends, diffusers, oblique cuts) in hot jet conditions.

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7 REFERENCES