Effect of geometrical parameters on high frequency performance of Rogowski coil for partial discharge measurements

Muhammad Shafiq * , G. Amjad Hussain, Lauri Kütt, Matti Lehtonen

Department of Electrical Engineering, School of Electrical Engineering, Aalto University, P.O. Box 13000, FI-00076 Aalto, Finland

1. Introduction

Increased reliability requirements of power supply networks and optimization of network assets are nowadays some of the major concerns of the distribution companies. Smart grid solutions implementing advanced substation and feeder automation technologies are proposed for achieving these goals. One example of such technologies is the active on-line condition monitoring of high voltage equipment. As the name refers, this is focusing on the real-time observation of the network components operation to detect the possible problems and the problematic components at an early stage.

A power distribution network is always exposed to various operational and environmental disturbances which in many cases cause interruption of supply [1]. Earth faults, short circuits and over-current are some of the most common types of faults in the network [2], with insulation degradation being one of the major causes of these faults. The insulation degradation process can be observed generally in two stages. The first stage is the pre-breakdown operation, starting with slow damage of the insulation followed by further insulation degradation with time. Second one is a post-breakdown condition when the complete failure of the insulation occurs, and the component is damaged beyond recovery. If an upcoming fault can be detected during the pre-breakdown operation, the faulty component could be repaired in advance, which would lead to significantly shorter power outage duration due to the faulty component.
In the pre-fault stage, the insulation damage of the power components produces events called partial discharges (PD). A PD is a momentary localized discharge which does not bridge the two electrodes. PD activity can be found in all major electrical components such as switchgear, transformers, rotating machines, overhead cover conductor lines and underground cables [3–5]. Each PD event generates low amplitude pulses of extremely short duration. These pulses can be observed as voltage and current pulses, superimposed on the mains voltage and current. Due to the very short rise time, the spectrum of such transients can reach well into the radio frequency region [6]. This also means that a sensor for detecting and accurately quantifying the pulses is required to have a wide operating bandwidth and high sensitivity.

Some major requirements of ideal high frequency pulse current measuring sensors can be listed as:

- Physical features: Cost effective, light weight, flexibility of installation.
- Measuring features: Suitable for online non-intrusive measurements, safety of the sensor and measuring system.
- Operational features: Higher sensitivity, wide bandwidth and linearity of operation.

A wide variety of current sensing equipment has been proposed for transient current measurements in [7]. The Rogowski coil has been considered to be one of the most favorable sensors with respect to the above requirements. Low- and medium-frequency current sensing in power systems and power electronics applications focuses on the design of the coil from a few amperes to a few hundred amperes, with bandwidth reaching up to a few MHz [8,9]. For applications such as PD detection and measurement, the design of the coil is focused on high frequencies up to tens to hundreds of MHz [8]. The above features for a Rogowski coil depend on geometrical parameters in one way or another. The literature on the design and modeling of Rogowski coils is available with some initial discussions about the impact of their major geometrical parameters [10–12]. Considering the application and installation locations or spaces, geometry of the coil needs to be changed for certain measuring requirements. These parameters significantly affect the key performance regarding the operation of the coil i.e., sensitivity and bandwidth [13,14].

The aim of this paper is to provide a more general and practical overview based on the experimental investigation of the performance of the sensor, with focus on the variation of key geometrical design parameters. The direct influence of each geometrical dimension of a wide-bandwidth Rogowski coil design on the electrical behavior of the sensor is presented. Various designs are compared with a reference design and a comparison is made for high frequency PD pulse measurements. The recommendations of the paper can be adopted as guidelines to identify the proper selection of the geometrical parameters during the design process of a Rogowski coil.

2. Induction current sensor principles

When a conductive loop is located in a time-varying magnetic field, Faraday’s law can be used to specify a relation between the time-variable magnetic flux $\Phi$ and electromotive force (emf) $e$ induced in the loop as

$$e = -\frac{d\Phi}{dt} \tag{1}$$

In a scalar form, the magnetic flux can be found as

$$\Phi = \int_A B \cos \theta \, da \tag{2}$$

where $B$ is the value of the magnetic flux density and $A$ is the area that the magnetic flux density is passing through, as shown in Fig. 1(a). Here the magnetic flux has been provided in the $x$, $y$ and $z$ axis components for clear presentation with a magnetic loop in a random position, described with a normal vector $\hat{a}_{\text{norm}}$. The magnetic flux is at a maximum when the angle $\alpha$ between the area’s normal vector $\hat{a}_{\text{norm}}$ and the magnetic flux density vector $B$ is zero.

A current carrying wire provides a magnetic field around it. A wire with a circular cross section will have a circular magnetic field, with the magnetic field density vector in the direction of the magnetic field tangent. This is visualized in Fig. 1(b), where a magnetic field $B$ is observed at a point $P$ near the wire. The resulting magnetic flux is perpendicular to the radius vector $r$. The magnetic field intensity $H$ and the magnetic field density $B$ at some defined point are in a reciprocal relation to the distance from the wire

$$B = \mu_a \cdot H = \frac{\mu_a \cdot I}{2\pi \cdot r} \tag{3}$$

where $\mu_a$ is the absolute magnetic permeability of the medium (for air $\mu_a = 4\pi \times 10^{-7}$ H/m). If $N$ identical loops are placed around a wire, all with the same distance from the wire and in a perpendicular position to the radius vector ($\theta = 90^\circ$), the sum of magnetic flux $\Phi_{\text{tot}}$ enclosed by all loops is:

$$\Phi_{\text{tot}} = \int_A B \cos \theta \, da = N \int_A \hat{a}_{\text{norm}} \cdot H \, da = N \mu_a \frac{I}{2\pi} \int_A \frac{1}{r} \, da \tag{4}$$

Current variation in time provides magnetic flux variation in time. Observing the loops connected in series

![Fig. 1. (a) Calculation of magnetic flux for a loop and (b) magnetic flux around a current-carrying wire.](image-url)
substituted into the same time-variable magnetic flux, the emf generated would be

\[ e = -\frac{d\Phi_{tot}}{dt} = -\frac{N\mu_0 I}{2\pi} \int_A \frac{1}{r} \, da = -\frac{d}{dt} \frac{N\mu_0 I}{2\pi} \int_A \frac{1}{r} \, da \]  

(5)

It can be assumed that the geometrical parameters will not vary with time. This divides Eq. (5) into the time-differential of current \( \frac{di}{dt} \) and the time-invariant quantities, constants and geometrical parameters as

\[ e = -M \frac{di}{dt} \]  

(6)

where

\[ M = \frac{N\mu_0}{2\pi} \int_A \frac{1}{r} \, da \]  

(7)

Here \( M \) is the mutual inductance of an inductive sensor. It is one of the key terms to determine the electrical behavior of inductive sensor coil.

3. Construction of Rogowski coil

The coils presented in this paper have been built in a laboratory workshop using a circular core for the purpose of PD measurements. The coil’s winding is constructed on a non-magnetic material of length \( l_c \), as shown in Fig. 2. Starting from End A of the core, the coil winding of \( N \) number of turns progresses towards End B of the core and after the last turn, a return wire is passed through the center of the core to End A of the core. The return wire serves as the return loop for the Rogowski coil. Using such a construction, both terminals of the coil winding are available at the same end (End A) of the coil. This makes the coil a flexible open end design, ready for fitting around a power line or component. During mounting, the ends A and B of the core are attached together.

Fig. 3 shows the normally installed (toroidal) operating form of the Rogowski coil in a loop shape. The geometry of the coil can be defined in terms of the following parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_o )</td>
<td>Outer diameter of coil</td>
</tr>
<tr>
<td>( d_i )</td>
<td>Inner diameter of coil</td>
</tr>
<tr>
<td>( d_{rc} )</td>
<td>Core diameter</td>
</tr>
<tr>
<td>( d_m )</td>
<td>Mean diameter of coil</td>
</tr>
<tr>
<td>( d_w )</td>
<td>Diameter of winding conductor</td>
</tr>
</tbody>
</table>

For accuracy of the measurement, the Rogowski coil should be mounted in such a way that the current carrying wire passes through the center of the sensor. This way the turns are located perpendicular to the current to be measured (see Fig. 4).

4. Characteristics depending on geometry of the Rogowski coil

As mentioned earlier, the electrical response of the coil model is determined by its geometrical parameters. The following overview is presented for the assessment of a coil’s most critical characteristics that are directly dependent on the coil geometry. Such characteristics are mutual inductance, self-resistance, self-inductance and self-capacitance of the Rogowski coil.

4.1. Mutual inductance

Mutual inductance is a decisive parameter for the assessment of a coil’s sensitivity. Its value depends on the shape of the cross section of the core used for the coil winding. The most common core cross-sections are circular, oval or rectangular. For different shapes of the coil core cross-section, different expressions are available to determine the mutual inductance. For these shapes, single turn based expressions [15–17] to find the mutual inductance are provided in Table 1. The formulas assume that all \( N \) loops have identical geometry and radial position from the wire being measured, which is held in the center of each type of coil. The measurement performance of these coil shapes for high frequency signal measurement has been presented in [18]. The circular cross section is
considered further in this paper for the design and analysis of Rogowski coils.

4.2. Self-resistance

The coil’s self-resistance is due to the resistance of the wire that is used for making the winding. The resistance depends on the wire cross-section area $A_w$, length of wire used $l_w$ and the material’s characteristic resistance.

The wire length of a single turn of the Rogowski coil depends on the turn pitch $p_w$ (Fig. 5) of the winding and the diameter of the core $d_{rc}$

$$l_w = N \sqrt{(\pi d_{rc})^2 + p_w^2}$$  \hspace{1cm} (8)

where $p_w = (l_i/N)$ for a winding with uniform turn density. The total coil wire resistance $R_c$ is

$$R_c = \frac{\rho l_c}{A_w} = \frac{\rho N \sqrt{(\pi d_{rc})^2 + p_w^2}}{A_w},$$  \hspace{1cm} (9)

where $\rho$ is the electrical resistivity of the conductor. For Rogowski coils with higher operating frequency range, the number of turns is generally low, therefore the length of wire used and also the resistance of the wire are rather small. In many situations the value of such resistance plays only an extremely small effect on the overall Rogowski coil performance.

4.3. Self-inductance

The inductance of a single circular loop $L_{loop}$ of wire can be found as [19]

$$L_{loop} = \mu_0 \frac{d_m}{2} \left[ \ln \left( \frac{8 \cdot d_m}{d_w} \right) - 2 \right]$$  \hspace{1cm} (10)

where $d_m$ is $(d_i + d_o)/2$. The inductance is dependent on the loop dimension and also the wire diameter. The Rogowski
The self-capacitance of a Rogowski coil with the construction specified above could be observed as the sum of two capacitance values. The first one is the turn-to-turn capacitance of the coil turns, present for each multi-turn inductor. The second is a specific capacitance that is due to the return wire in the center of the winding. In the latter case, the capacitance is formed between the turns of the coil and the return winding. The geometrical formation of the two capacitances is presented in Fig. 6, where \( C_t \) marks the turn-to-turn capacitance and \( C_r \) stands for the return wire capacitance.

The turn-to-return capacitance has been suggested to be calculated as a coaxial system, with the winding perimeter as the outer electrode and the return wire as the center electrode, see Eq. (12)[20].

\[
C_r = \frac{4\pi \varepsilon_0 \varepsilon_r (d_o + d_i)}{\log(d_o + d_i)/(d_o - d_i)}
\]

The turn-to-turn capacitance can be calculated as [9]:

\[
C_t = \frac{\pi \varepsilon_0 \ell_w}{(N - 1) \log \left( \frac{d_i}{d_o} \right) + \sqrt{\left( \frac{d_o}{d_i} \right)^2 - 1}}
\]

This assumes that the capacitance between turns would be the same as wire rings (turns) placed side-by-side, making up a chain of series-connected capacitors. Therefore equivalent turn-to-turn capacitance for a coil with the higher number of turns would become very small and can be neglected.

There has been a variety of formulas in the literature (including the above mentioned formulas in this section) presented with different approaches to obtain the capacitance and inductance values for Rogowski coils based on their geometry. These formulas can be used for basic assessment of these parameters. However, it turns out that the formulas are not very precise when accurate results are needed. Especially difficult are the coil capacitance and inductance calculations [12]. A more practical approach is presented in Appendix A [21] and is used to determine the capacitance and inductance accurately. The advantage of this measurement based methodology is that the coil measuring system is exposed to the measurement environment and the high frequency components of measured signals, which provide more reliable parameters for the coil.

5. Equivalent circuit and high-frequency operation

The electrical model of a Rogowski coil can be represented as a lumped parameter model as shown in Fig. 7. Here \( R_c \) and \( L_c \) are the self-resistance and self-inductance, whereas \( C_{cs} \) is the combined system capacitance referring to the sum of the self-capacitance of the coil \( C_c \) and the measuring system \( C_m \).

\[
C_{cs} = C_c + C_m
\]

Considering the R–L–C second order circuit in Fig. 7, in response to the primary current \( i_p(t) \), the output voltage \( V_o(t) \) of the Rogowski coil can be determined by the transfer function (in s-domain) given as

\[
V_o(s) = \frac{1}{s^2 + \frac{1}{LC_c} + \frac{1}{RC_c}} \frac{M_v}{s} + \frac{1}{\ell_w} \frac{M_v I_p(s)}{s}
\]

where \( R_m \) is the measuring resistance (in this case, oscilloscope) which is acting as the terminating resistance across the terminal of Rogowski coil. The transfer function based on the lumped parameter model can be used to determine the sensitivity and bandwidth of the designed coil, which are the basic requirements of the design for particular applications. Depending upon the value of mutual inductance \( M_c \), the sensitivity can be expressed by (14), while the resonance frequency of the coil is expressed as

\[
f_r = \frac{1}{2\pi\sqrt{L_c C_{cs}}}
\]

Observing the Rogowski coil operation in terms of (6), the output voltage of the coil is the time-differential of the measured current. Plots 1 and 2 in Fig. 8 present the undamped and critically damped modes of operation for coil ‘A’, respectively. The undamped response (plot 1) shows the resonance at 60.7 MHz for coil ‘A’, while a
suitable terminating resistance is added across the coil output to damp the oscillating response. During this operation the transfer rate rises by a factor of 10 for frequency increase of 10 times. Solving (14) it becomes evident that this trend is valid only up to the resonant frequency. In addition, if the resonance is not damped (plot 1 in Fig. 8), a significant increase in transfer rate can be seen for the signal component having the resonant frequency. A detailed comparison of different modes of operation for a Rogowski coil is presented in [22].

A pulsed signal consists of a large number of high frequency components. This means that the sensors should have a wide operating bandwidth to capture the pulse accurately. Bandwidth in this context is the frequency range, where the coil is providing a steady and relatively high value of sensitivity (i.e., magnitude transfer) with minimum output phase shift (preferably 0°) for all frequency components measured. Such operation can be achieved, if the output of a critically the damped sensor (where damping rate for oscillations at the resonant frequency is 1) is integrated (Fig. 8, plots 3 and 5).

In this work, the sensors observed are considered to operate in a critically damped mode. The output of the sensor, in ideal conditions, would be a differential of the measured current. In the Bode plot in Fig. 8, this is represented as plot 2, with the phase transfer at +90° (plot 4) reaching from low frequencies up to resonant frequency. By integrating the output voltage signal of the sensor (see Fig. 8, plots 2 and 4), the pulse-measuring criteria are similarly satisfied from very low frequency up to the resonant frequency. It can be seen that for frequencies higher than the resonant frequency, the magnitude transfer rate actually drops with a rate similar to the second order filter after the resonant point. Therefore, this can be considered to be the sensor’s higher cut-off frequency, meaning the highest frequency which the sensor could operate at, would be limited by the resonant frequency. This is why in the following analysis, resonant frequency is considered to be the critical property for the Rogowski coil. A higher resonant frequency also means a higher upper operating frequency, which is a very beneficial feature for measuring the small-magnitude PD pulses with extremely short duration.

The Rogowski coil sensor will most likely also have a low cut-off frequency, as the differential of the measured current \(\frac{dp}{dt}\) will become so low that the sensor output voltage corresponding to a differential of this magnitude will be lower than the noise present in the output of the sensor. However, the lower cut-off frequency will be dependent not only on the sensor itself but also on the other components of the measurement system (for example, digital-to-analog converter input noise level, etc.). As in the present context the direct effects of the Rogowski sensor geometry are discussed, the lower (cut-off) operating frequency will not be considered. Instead, it will be assumed that a sensor has steady response from frequency of 1000 Hz upwards, which is still sufficient for cutting out the mains frequency current signal.

### 6. Experimental setup for evaluation of Rogowski coils

For this work, six different coils, with geometrical data provided in Table 2, were prepared in the laboratory. Coil “A” is considered to be the reference coil, to which all other coil designs are compared. The reference coil has been used in several previous measurement scenarios and detailed descriptions of its design and performance have been reported in [21], [23]. The coils used in this analysis were custom-made in the laboratory. The difference in designs of the coils presented in Table 2 are due to variation in the parameters, core diameter \(d_{in}\), coil diameter \(d_m\), number of turns \(N\), wire diameter \(d_w\), and type of return loop.

In this paper the behavior of Rogowski coils with different design parameters is compared with respect to their sensitivity and bandwidth. The sensitivity is evaluated by the time domain (amplitude) response of the coil, while the bandwidth analysis is performed by observing the sensor resonant frequency.

To determine the amplitude and resonant frequency of the coils, each coil under test was set to measure a sharp current pulse from a pulse generator (see Fig. 9). Fig. 10 presents the transient pulse measured by Rogowski coil “A”. The output voltage of the coil is measured by a digital storage oscilloscope (DSO) at a sampling time of \(4 \times 10^{-10}\) s. The signal is captured across the open terminal of the coil, with \(R_m = 1\, \text{MΩ}\), which is the input resistance of the oscilloscope. The use of an active differential probe

![Fig. 9. Experimental setup for measurement of PD pulse.](image)

<table>
<thead>
<tr>
<th>Coil type</th>
<th>(d_i) (mm)</th>
<th>(d_o) (mm)</th>
<th>(d_m) (mm)</th>
<th>(d_{in}) (mm)</th>
<th>(d_w) (mm)</th>
<th>(N)</th>
<th>Parameter variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>155.0</td>
<td>131.0</td>
<td>12.0</td>
<td>143.0</td>
<td>0.85</td>
<td>30</td>
<td>Reference coil</td>
</tr>
<tr>
<td>B</td>
<td>160.0</td>
<td>126.0</td>
<td>17.0</td>
<td>143.0</td>
<td>0.85</td>
<td>30</td>
<td>Core diameter</td>
</tr>
<tr>
<td>C</td>
<td>107.6</td>
<td>83.6</td>
<td>12.0</td>
<td>95.6</td>
<td>0.85</td>
<td>30</td>
<td>Coil diameter</td>
</tr>
<tr>
<td>D</td>
<td>155.0</td>
<td>131.0</td>
<td>12.0</td>
<td>143.0</td>
<td>0.85</td>
<td>60</td>
<td>Number of turns</td>
</tr>
<tr>
<td>E</td>
<td>155.0</td>
<td>131.0</td>
<td>12.0</td>
<td>143.0</td>
<td>0.85</td>
<td>30</td>
<td>Wire diameter</td>
</tr>
<tr>
<td>F</td>
<td>155.0</td>
<td>131.0</td>
<td>12.0</td>
<td>143.0</td>
<td>0.85</td>
<td>60</td>
<td>Return winding</td>
</tr>
</tbody>
</table>
(ADP) reduces the loop currents and common voltages across the output of the Rogowski coil. Due to low loading, the coil oscillations at its natural frequency are not damped. The pulse in red shown in Fig. 10(a) is the primary signal measured by a commercial high frequency current transformer (HFCT). The oscillating (blue) signal is the measured output of the Rogowski coil. The first peak of the measured signal $V_o$ presents the amplitude of the output [23]. The resonant frequency is identified by performing a Fourier transform on the output signal, as presented in Fig. 10(b).

### 7. Effect of geometrical parameters on the performance of the Rogowski coil

A comparison of the given coil characteristics is presented in this section keeping in view the practical mechanical and electrical consideration of the designs.

#### 7.1. Diameter of core ($d_{rc}$)

A larger $d_{rc}$ results in an increased cross section $A_c$ of the core, and thus a higher mutual inductance value as can be seen. Such behavior of the mutual inductance can be assessed by considering the expression for calculating $M_c$ in Table 1. The sensitivity is directly linked to the value of the mutual inductance (see Eq. (14)) and therefore the sensitivity of the coil is increased. Higher sensitivity means the ability to measure very small PDs that are occurring just at the initial stage of the insulation defects. On the other hand, as can be seen by (11), the increased coil winding loop area also means a higher inductance value. This can also be seen in the comparison of the resonance frequencies (see Table 4). A Coil with a larger turn area has a significantly lower bandwidth. The measured results show that sensitivity increase by a factor of almost two times would lead to a decrease in the resonant frequency of 16%.

The increased diameter of the core may lead to some practical issues during installation. When using a flexible core material for the coil, a larger $d_{rc}$ makes it more difficult to bend or it gets de-shaped. For a proper bend of the coil increase in the diameter of the coil $d_m$ would be required which results in increased space requirements for the installation. Substations usually do not have abundant space available for the sensor mounting and cables are almost impossible to bend. For mounting the coil, depending upon the stiffness of the core material, a suitable ratio of $d_{rc}$ to $d_m$ can be established to enhance measuring compatibility.

#### 7.2. Diameter of coil ($d_m$)

A Rogowski coil would probably be prepared with as small dimensions as possible to lower the quantities of materials required, which reduces the manufacturing expenses. The inner diameter of the coil ($d_i$) is selected so

### Table 3

<table>
<thead>
<tr>
<th>Coil type</th>
<th>Capacitance (pF)</th>
<th>Inductance (µH)</th>
<th>Mutual inductance (nH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5.7</td>
<td>1.20</td>
<td>9.51</td>
</tr>
<tr>
<td>B</td>
<td>6.5</td>
<td>1.51</td>
<td>19.1</td>
</tr>
<tr>
<td>C</td>
<td>4.1</td>
<td>1.50</td>
<td>14.3</td>
</tr>
<tr>
<td>D</td>
<td>8.2</td>
<td>2.33</td>
<td>19.0</td>
</tr>
<tr>
<td>E</td>
<td>4.9</td>
<td>1.36</td>
<td>9.51</td>
</tr>
<tr>
<td>F</td>
<td>15.2</td>
<td>2.33</td>
<td>19.0</td>
</tr>
</tbody>
</table>

### Table 4

<table>
<thead>
<tr>
<th>Coil type</th>
<th>$d_{rc}$ (mm)</th>
<th>Resonant frequency (MHz)</th>
<th>Sensitivity (mV/A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12</td>
<td>60.7</td>
<td>9.51</td>
</tr>
<tr>
<td>B</td>
<td>17</td>
<td>50.8</td>
<td>19.1</td>
</tr>
</tbody>
</table>
that it could be assembled easily around the primary current carrying conductor.

A critical factor for the application is the level of electrical insulation. For higher voltages across the line that is being measured by the Rogowski coil, the core cannot be placed closer to the HV conductor than some specified distance. This is to guarantee that the electric field strength is less than the dielectric breakdown strength between the line and the winding of the coil. The dielectric strength (~3 MV/m for air) of the insulation material defines the thickness of insulation to be used. Based on practical experience, the designed coils can be used up to a few kilovolts with minimum insulation. However, for the higher voltages in a distribution network, the insulation has to be capable of providing safe and reliable operation during normal as well as faulty voltage levels. The problem can be addressed by providing additional insulation, either using better insulation materials or increasing the width of insulation. The first option increases the cost of manufacturing of the Rogowski coil. The second alternative increases the weight but also requires an increase in the outer diameter of Rogowski coil. For flexible core coils, the material should be soft enough to avoid any problem with bending.

A smaller diameter however helps to keep the coil closer around the power line wire. This would increase the sensitivity of the coil. From the aspects of bandwidth, it can be seen that decreasing the coil’s diameter will increase the coil natural resonant frequency (Table 5). This is due to a decrease in the coil inductance, which can be seen from Eq. (11). Considering Eq. (12), the capacitance will be decreased slightly. Finally, the measurement parameters are identified as shown in Table 3, with a reciprocal variation in the inductance and capacitance of the coil. Reducing the \( d_w \) by 50% of the original diameter results in a healthy increase of sensitivity by two times, as the winding sensing parts are now closer to the current carrying wire. At the same time, a 13% increase in the resonant frequency is seen as well.

Transients tend to ground through any closest ground path. Therefore, a Rogowski coil can be installed either on live phases or on ground terminals, in order to measure transient pulses. Installation on the grounding conductor (cable screening) does not require the higher insulation levels and Rogowski sensors designed for grounding wires can have significantly lower insulation levels, thus also smaller diameter.

### 7.3. Number of turns (N)

Sensitivity has a linear relation to the number of turns. Adding turns to the coil increases the value of mutual and self inductances as can be clearly seen from Table 1 and Eq. (11), respectively. Therefore, if higher sensitivity is required, increasing the number of turns could be one of the easiest ways to accomplish this.

A higher number of turns will once again bring higher inductance value. The inductance increase is quite high and this can be seen in the significant decrease of the resonant frequency (see Table 6). Based on these measured results, for a sensitivity increase of two times, the resonant frequency is decreased by approximately 33%. This means a greater decrease in the resonant frequency than has been presented in the case of increasing the core diameter. There are in fact only a few reasons on the mechanical side to keep the number of turns at a defined level. Therefore the number of turns is directly dependent on the electrical requirements.

### 7.4. Diameter of wire (\( d_w \))

The requirement of the winding wire is to provide good mechanical strength, suitable weight and low electrical resistance. A larger diameter would most likely be preferred for the lower resistance and higher mechanical strength. A change in wire diameter does not significantly affect the sensitivity. With high frequency operation, the increased resistance of the wire is more likely due to the skin effect. This increase in resistance does not, however, have any major effect on the operation of the Rogowski coil.

From Table 3 it can be seen that variation in the wire diameter will cause a minor change in the coil inductance and coil self-capacitance values. When the wire diameter is decreased by 24%, the inductance increases by 13%. At the same time, due to the smaller outer perimeter of the conductor, the self-capacitance value would decrease by 18%. As a result, the resonant frequency would be slightly higher, as shown in Table 7.

### 7.5. Return loop

With each successive turn in the toroidal helical coil winding, the incremental pitch between consecutive turns sums up to create an additional undesirable circumferential loop. This loop is sensitive to magnetic fields that do not originate from the current passing through the primary current wire. On the contrary, the return loop position is

---

**Table 5**

Comparison of Rogowski coils with different coil diameters.

<table>
<thead>
<tr>
<th>Coil type</th>
<th>( d_m ) (mm)</th>
<th>Resonant frequency (MHz)</th>
<th>Sensitivity (mV/A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>143</td>
<td>60.7</td>
<td>9.51</td>
</tr>
<tr>
<td>C</td>
<td>95.6</td>
<td>64.1</td>
<td>14.3</td>
</tr>
</tbody>
</table>

**Table 6**

Comparison of Rogowski coils with different number of turns.

<table>
<thead>
<tr>
<th>Coil type</th>
<th>( N )</th>
<th>Resonant frequency (MHz)</th>
<th>Sensitivity (mV/A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>30</td>
<td>60.7</td>
<td>9.51</td>
</tr>
<tr>
<td>D</td>
<td>60</td>
<td>36.4</td>
<td>19.0</td>
</tr>
</tbody>
</table>

**Table 7**

Comparison of Rogowski coils with different winding wire diameters.

<table>
<thead>
<tr>
<th>Coil type</th>
<th>( d_w ) (mm)</th>
<th>Resonant frequency (MHz)</th>
<th>Sensitivity (mV/A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.85</td>
<td>60.7</td>
<td>9.51</td>
</tr>
<tr>
<td>E</td>
<td>0.65</td>
<td>61.6</td>
<td>9.51</td>
</tr>
</tbody>
</table>
normal to the axis of the coil and thus insensitive to the magnetic fields produced by the primary conductor (\(\cos \frac{90}{C} = 0\) in Eq. (2)). The presence of such a natural loop would make the Rogowski coil sensitive to the magnetic fields produced by interferences, radio waves and other electromagnetic noise. To eliminate the effect of the loop as a source of noise, an electrically opposite return loop is made through the center of the winding core to cancel the magnetically induced effect. Generally, the center of the core (center of the coil winding) is recommended for the return loop by most researchers. However, there are reservations regarding the radial position of the return loop. It has been identified that the return loop area with a wire passing through the center of the winding is less than the circumferential (effective) loop area, which causes imperfect cancellation of the external magnetic field [10]. For a larger diameter core the difference between the above two types of loop area is greater. The behavior of cancellation effect of the return loop versus the diameter of the core is described in Appendix A.2.

An alternative for the compensation of the return loop is to have a return winding loop as shown in Fig. 11. The forward and return windings have to be identical regarding the number of turns, turn spacing and uniformity. For the analysis and comparison of the return winding, a coil with 30 turns of forward winding added with 30 identical turns of backward winding was prepared. Considering the total number of turns, it was compared to a coil of 60 turns with a return loop. The characteristic parameter measurement results are reported in Table 8.

The constructed return-winding Rogowski coil is a rather unprecedented design. However it has been observed that it does not provide advantages to the design with reference to sensitivity and bandwidth. It can be seen that the dimensions and number of turns are the same for both the return winding and the return loop type Rogowski coils, therefore the inductance has practically the same value, conforming to the expectation. The self-capacitance, however, is 85% higher. The reason for this is likely to be the higher potential gradients of the wires close together added to the fact that at some points the wires of the forward and return windings are overlapping.

A Rogowski coil with a return winding is likely to offer higher external noise cancellation compared to a coil with return loop. This is due to the symmetrical construction of the forward and return windings, whereas a return loop differs from the winding loop to a great extent.

### 8. Recommendations

The outcomes of the experimental analysis evaluated in the previous section, regarding variation in the dimensions of the Rogowski coil, are presented in Table 9. The comparison between the given alternatives is presented in terms of proportional changes. Considering the degree of change in the respective parameters (with reference to coil A) and comparing the effects on sensitivity and resonant frequency of the coils, a suitable trade-off should be made to make an optimal selection of the coil design.

The general outlines for making a coil with good sensitivity and wide bandwidth can be summarized as follows:

(i) As seen in Table 9, decreasing the coil diameter \(d_{cm}\) is the most favorable alternative for improved performance of the coil. Both sensitivity and bandwidth increase by reducing the coil diameter. Additional advantages are the smaller size, lower weight and a greater possibility that the under test line will be placed in the center of the Rogowski coil.

(ii) The core diameter seems to be the second preferred option to improve the electrical performance. The proportion of the change in diameter and its effect

<table>
<thead>
<tr>
<th>Table 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comparison of Rogowski coils with different types of return loops.</td>
</tr>
<tr>
<td>Coil type</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysis of the effects of changes in geometry of the Rogowski coil.</td>
</tr>
<tr>
<td>Name of parameter</td>
</tr>
<tr>
<td>Core diameter</td>
</tr>
<tr>
<td>Coil diameter</td>
</tr>
<tr>
<td>Number of turns</td>
</tr>
<tr>
<td>Wire diameter</td>
</tr>
<tr>
<td>Type of winding</td>
</tr>
</tbody>
</table>

R.L – return loop, R.W – return winding.
on the performance can be seen in Table 9. It should
be kept in mind while adopting this alternative that
the core diameter should only be increased to the
extent that it will not be de-shaped during
installation.
(iii) Adding the number of turns to increase the sensitiv-
ity would not be recommended, as it decreases the
operating bandwidth significantly. The additional
disadvantage is the increased weight.
(iv) During the analysis, it has been observed that varia-
tion in wire diameter does not have much effect on
the electrical response. Mechanical strength and
the weight of the coil might be the main criteria to
select the wire diameter. Manufacturing price may
influence the choice of the conductor diameter. The
conductor diameter is more influential for measure-
ments at the mains frequency, as such low

frequency measurements (50 Hz) would demand a
higher number of turns. Although the expected
mains currents measured can be in the range of hun-
dreds of amperes, the low rate of the time-differen-
tial value of the current would mean a low output.
For a sensor with a very large number of turns, the
wire diameter would influence the sensor’s mass
as well as its price to a higher extent.
(v) The return loop is preferable over the return wind-
ing. The return winding provides basically the same
sensitivity, however there is a significant increase in
capacitance, which results in significant reduction in
the bandwidth of the coil. Rogowski coil required for
a harsh and noisy environment, typically in an
industrial plant, where surrounding noise is
expected to disturb the measurements, return wind-
ing is recommended instead of return wire loop. The
return winding design is more efficient when a lar-
ger diameter of coil is required.

9. Rogowski coil as current measuring sensor

For a Rogowski coil acting as a current measuring de-
vice, three cascaded components should be considered as
a set: the Rogowski coil head, damping components and
integator. The effect of the geometrical parameters is observed (in previous sections) for the sensing element of the coil, which is the Rogowski coil head. The latter two components are not directly affected by the geometry of the coil. These components deal with the signal processing of the output signal obtained by the Rogowski coil head. The sequential procedure of the development of Rogowski coil as PD current measuring sensor is provided in the flow chart shown in Fig. 12, and has been implemented in [21].

The Rogowski coil head senses the high frequency transient primary current as shown in Fig. 10. As can be seen from Eq. (14), the Rogowski coil has a second order transfer function which can have an oscillatory transient response. The output oscillations introduced due to the sensor's characteristics would need to be damped by using an external terminating resistance $R_t$. The damping resistance (connected as a terminating resistance $R_t$) is determined by the characteristic impedance $Z_c$ of the coil, which is expressed as [24]:

$$Z_c = \sqrt{\frac{L_c}{C_c}} \tag{16}$$

As given in Eq. (6), the output $V_o(t)$ of a Rogowski coil is the differential of the primary current $i_p(t)$, therefore the damped signal is integrated to extract the primary signal wave shape. The output of the integrator is in reciprocal relation to the input value by the mutual inductance $M$. Using the value of the mutual inductance, the sensitivity of the measurement system $K$ can be found. However, it is recommended to calibrate the sensor and adjust the sensitivity factor according to the measured output.

10. Conclusions

In this paper the measurement performance of the Rogowski coils for a high frequency current pulse is evaluated as a function of its geometrical characteristics. Considering a PD current pulse, different models of Rogowski coils have been investigated on the basis of key performance indicators, i.e., sensitivity and bandwidth. Higher sensitivity and bandwidth are usually the requirements for PD applications. A quantified approach has been presented to ascertain the comparative effectiveness of the different design parameters of the coil. The comparison is based on experimental investigation, which improves the reliability of results by taking into account the operational factors. The analysis and guidelines are presented with a simple approach, which should be useful when trading off the benefits for better mechanical and electrical design of the Rogowski coil. An optimized design can ensure the accurate measurement of PD signals, which provides a true interpretation of the insulation condition of power components.

Acknowledgements

The research is supported by the Smart Grid and Energy Market research program, Cleen Oy, Finland. The authors highly appreciate the valuable and interesting discussions with Dr. Petri Hyvönen and Dr. John Millar (department of Electrical Engineering, Aalto University).

Appendix A

A.1. Parameter identification

The Resonant frequency is a clear reflection of the electromagnetic components of an induction sensor system. A simple technique for comparing the resonant frequencies of Rogowski coil is used for different known capacitors connected across the terminals [21]. The experimental setup made for the identification of the parameters is shown in Fig. 13(a). The targeted parameters for identification in the Rogowski coil system are $L_c$ (self-inductance), $C_c$ (self-capacitance) and $C_p$.

In the double stage test reported below $C_c$ and $C_p$ are determined. The tests are carried out by observing the Rogowski coil output waveforms and the oscillation frequency for different known testing capacitors $C_T$, which are connected across the terminals of the coil as shown in Fig. 13(b). Two sets of measurements are planned to develop the mathematical equations that determine two unknown capacitances, $C_c$ and $C_p$ as follows:

A.1.1. First set of measurements

Two measurements are carried out by first connecting capacitor $C_{T1}$, and on the second round $C_{T2}$ across coil. The response of coil system is captured using a single ADP. The two resonance frequencies $f_{11}$ and $f_{12}$ respectively, are obtained by performing fast Fourier transform (FFT) of the captured response (see Fig. 10). The resonance frequency found by measurements can be expressed as

$$f_{11} = \frac{1}{2\pi \sqrt{L_{11}(C_c + C_T + C_p)}} \tag{17}$$

Comparing the obtained resonance frequencies

$$f_{11} = \frac{L_{11}}{\sqrt{C_c + C_T + C_{T1}}} \Rightarrow C_c + C_T + C_{T1} = (C_c + C_T + C_{T1}) \cdot \left(\frac{f_{11}}{f_{12}}\right) \tag{18}$$

This finally gives

$$C_c + C_p = \frac{C_{T1} \left(\frac{f_{11}}{f_{12}}\right)^2 - C_{T2}}{1 - \left(\frac{f_{11}}{f_{12}}\right)^2} \tag{19}$$

A.1.2. Second set of measurements

Two similar measurements are taken by connecting capacitors $C_{T1}$ and $C_{T2}$. In this test, the response of the coil system is captured using two identical ADPs in parallel, which provide resonant frequencies $f_{21}$ and $f_{22}$ respectively. These results provide essentially the same formulas but now two probe capacitances $C_T$ are in parallel, thus

$$f_{21} = \frac{L_{11}}{\sqrt{C_c + 2C_p + C_{T1}}} \Rightarrow C_c + 2C_p + C_{T1} = (C_c + 2C_p + C_{T1}) \cdot \left(\frac{f_{21}}{f_{22}}\right)^2 \tag{20}$$
\[ C_0 + 2C_p = \frac{C_{T1} \left( \frac{m}{L} \right)^2 - C_{T2}}{1 - \left( \frac{m}{L} \right)^2} \quad (21) \]

From (19) and (21) values for \( C_c \) and \( C_p \) are calculated. The inductance values can be calculated as

\[ L_{11} = \frac{1}{2\pi(C_c + C_p + C_{T1})f_{T1}^2} \quad (22) \]

\[ L_{12} = \frac{1}{2\pi(C_c + C_p + C_{T2})f_{T2}^2} \quad (23) \]

where \( L_{11} \) and \( L_{12} \) are the effective inductance values in the first set of measurements. A negligible difference is seen when calculating \( L_{21} \) and \( L_{22} \) using the second set of measurements. The average of results can be used as the coil inductance \( L_c \).

A.2. Noise cancellation effect of return loop

If the coil encounters the uniform external magnetic field directed normal to effective plane of the ring of the core, plane area of the effective (circular) loop of the winding \( A_{EL} \) is given by [10]:

\[ A_{EL} = \frac{\pi d_e^2}{4} = \frac{\pi}{4} \left( \frac{3d_o^2 + 3d_i^2 + 2d_i d_o}{32} \right) \quad (24) \]

where \( d_c \) is the effective (circular) diameter of the incremental loop, \( d_i \) is the inner diameter and \( d_o \) is the outer diameter of the core respectively, shown according to their respective radius in Fig. 14(a).

In the case of a return loop, the loop is at a radial position \( r_{m} \). Here \( r_{m} = (r_{1} + r_{0})/2 \) and the plane area of the return loop \( A_{RL} \) is calculated as

\[ A_{RL} = \frac{\pi d_e^2}{4} = \frac{\pi}{4} \left( \frac{d_i^2 + 2d_i d_o + d_o^2}{8} \right) \quad (25) \]

It can be noticed from (12) and (13) that \( A_{RL} < A_{EL} \), which reveals that the emf induced due to the external magnetic field sensed by effective loop area \( A_{EL} \) is not fully compensated by the return loop area \( A_{RL} \). Another factor affecting the percentage error of this compensation depends upon the ratio \( d_d/d_i \). A graphical analysis is made by assuming \( d_i = 10 \text{ cm} \) and \( d_o = 12–20 \text{ cm} \). The coil's cross-section \( d_{rc} \) varies from 2 to 10 cm in Fig. 14(b).

References